STUDY OF A MARINE NUCLEAR PROPULSION PLANT THROUGH COMPUTER SIMULATION

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THESIS

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ABSTRACT

The dynamics of a proposed marine nuclear propulsion plant, under the condition of sudden changes in load, are studied by both an analog and digital simulation. Two inherent reactor characteristics, internal reactivity feedback and the existance of delayed neutron emitters, which effect power plant stability and controllability are considered in detail.

Comparison of the simulation results with practical data indicates that the simulation represented the dynamics of a marine nuclear power plant with a sufficient degree of accuracy to be extremely useful in the study of marine nuclear power plants.

Evaluation of the simulation results indicate that the proposed power plant is inherently stable when operating in the normal region. The desirability of control schemes are discussed and a control scheme utilizing a constant average coolant temperature program is implemented. This external control scheme significantly improved the response of the system.

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TABLE OF SYMBOLS AND ABBREVIATIONS

SYMBOL	DEFINITION
A	Area
A_1	Constant of proportionality relating power and neutron density.
c	Density of delay neutron emitters
$c(0)$	Density of delayed neutron emitters at time $t = 0$
c_c	Specific heat of the coolant
c_f	Specific heat of the fuel
c_s	Specific heat of the steam
D	Normalized density of delayed neutron emitters.
E	Arbitrary constant
k	Excess reactivity
k_1	Coefficient of heat transfer
k_2	Coefficient of heat transfer
K	Excess reactivity in dollar units
K_R	Reactivity associated with rod position
K_M	Motor constant
K_T	Throttle constant
ℓ	Prompt neutron lifetime
m_c	Mass flow rate of the coolant
m_f	Mass of the fuel
m_s	Mass flow rate of the steam

SYMBOL	DEFINITION
n	Neutron density
$n(0)$	Neutron density at time $t = 0$
N	Normalized neutron density
P	Power
q	Heat transferred
q_c	Heat transferred to the coolant
q_f	Heat transferred to the fuel
q_s	Heat transferred to the steam
r	Radius
T_{AV}	Average coolant temperature
T_C	Cold leg temperature
T_F	Fuel temperature
T_H	Hot leg temperature
T_{BI}	Boiler inlet temperature
T_{BO}	Boiler outlet temperature
T_B	Average temperature in the primary side of the boiler
T_M	Mean temperature difference between T_S and T_B
T_S	Steam temperature
T	Temperature
x	Analog scaling factor. There are several different analog scaling factors which are denoted by subscripts
α_c	Reactivity feedback coefficient associated with coolant temperature
α_F	Reactivity feedback coefficient associated with fuel temperature

SYMBOL	DEFINITION
τ	Time constant. There are several different time constants denoted by subscripts
ψ	Fraction of normal load
β	Delayed neutron fraction
λ	Decay constant for the delayed neutrons

All variables with a dot (•) above them represent the time derivative of these variables. All variables with a bar (-) above them represent analog computer variables.

ACKNOWLEDGEMENT

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I. INTRODUCTION

Nuclear reactor power plants for use in marine application differ from stationary power plants in that they must respond quickly to power demands which may vary throughout a wide range in a relatively short period of time. There are maneuvering situations where the shipboard power plant may be required to respond to load changes of up to 50 percent in a few seconds. In addition to the operational differences, the mobile nature of the nuclear power ship coupled with the size and weight limitations which must be imposed upon the reactor shielding require that safety be a primary consideration in design and control of the marine power plant.

There are several references [Harrer 1963 and Glasstone and M. C. Edlund 1952] that treat the nuclear reactor and nuclear reactor control from the standpoint of the reactor operating at constant output power. Schultz [Ref. 3] discusses the problems of reactor control from the standpoint of a reactor operating with varying power demands as well as when operating under steady power out conditions. King ~~[Ref. 4]~~ considers the problems associated with nuclear power systems largely from the position of a mechanical engineer.

This thesis investigates a marine nuclear power plant and is intended as an initial look into the dynamics of this type of propulsion system. The system dynamic equations are developed and the resulting fundamental balance equations are

simplified by invoking reasonable first approximations. Initially a power plant without an external controller is studied. After the initial investigation a control scheme is formulated and implemented. The plant response with controller is compared with the plant without controller.

The power plant, represented in block diagram form in Figure (1-1), utilizes a pressurized water reactor fueled with low enriched uranium dioxide, heat exchanger for the conversion of reactor power to steam and a turbine which is represented by the changing load. The reactor and one side of the

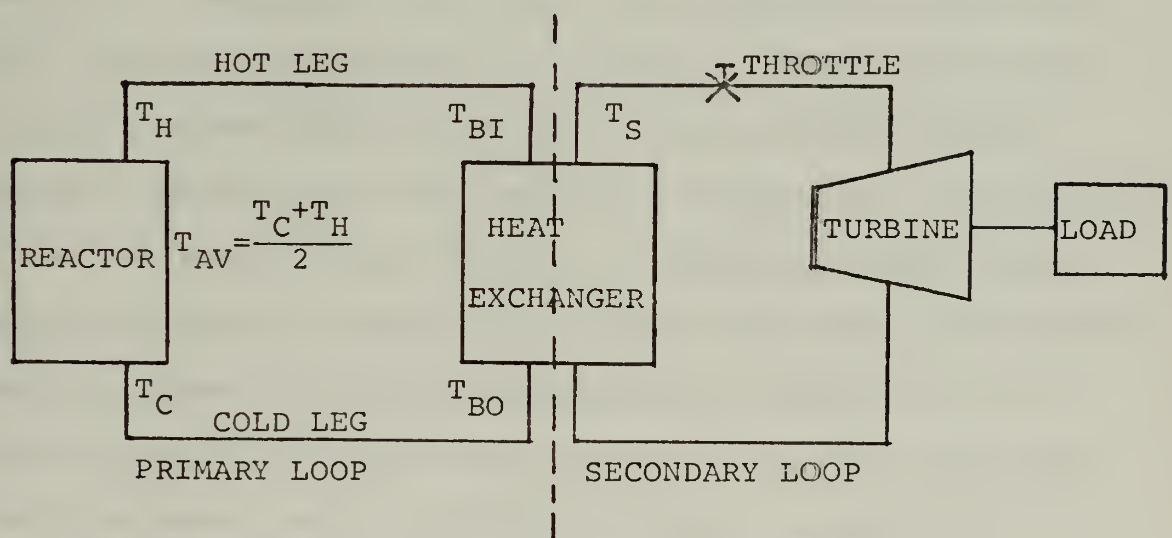


Fig. (1-1). BLOCK DIAGRAM OF BASIC NUCLEAR POWER PLANT

heat exchanger represent the primary loop while the other side of the heat exchanger and turbine represent the secondary loop in the system.

The system, although hypothetical, is similar to the plant on board N. S. Savannah. Many of the initial (before load changes) temperatures and coefficients closely approximate those of Savannah ~~[5, 6, 7]~~. In order to show the validity of the mathematical model used in this study, a qualitative comparison between the response of the simulation and both those of Savannah and a proposed marine power plant is used [8].

NO

The organization of this thesis follows the format as described below: Chapter (2) contains a development of the mathematical model used for the evaluation of system performance; Chapter (3) contains a general discussion of two inherent reactor characteristics, delayed neutrons and internal reactivity feedback, that affect the stability of the power plant; Chapter (4) presents an analysis of the analog and digital computer simulations of the power plant without external control and also contains a comparison of the results of the two simulations; Chapter (5) discusses some general aspects of nuclear power plant control and shows the specific control scheme used in this evaluation; Chapter (6) is a discussion of the conclusions arrived at by this study and also discusses areas of further interest. Appendices A through D supplement these chapters.

NO

II. MATHEMATICAL MODEL

The mathematical model for the plant under consideration is developed by obtaining the model for each system component individually. The individual models together with the values of the specific system parameters are summarized at the end of this chapter. During the course of the development, it is necessary to make several simplifying assumptions. These assumptions introduce some inaccuracies in the system model. "However, the errors involved in these assumptions are usually less than the amount of uncertainty in the engineering value of the coefficients used." [3] The generalized equations given below are listed in [Ref. 9].

A. REACTOR KINETICS

The point model kinetic equations of a chain reacting nuclear pile have been developed in the literature [10, 11] and shall be used for the starting point in the development of the plant model to be studied.

$$\dot{n} = \left(\frac{\delta k - \beta}{\ell}\right) n + \sum_{i=1}^{6} \lambda_i c_i \tag{2-1}$$

$$\dot{c}_i = \frac{\beta_i}{\ell} n - \lambda_i c_i \tag{2-2}$$

Equations (2-1) and (2-2) are state equations describing a nuclear reactor operating at some steady state power condition. The variables n, c, and δk represent neutron density, density

of delayed neutron emitters and excess reactivity. The other terms are constant coefficients described ~~on page 7~~.

The indexed terms in equation (2-1) and (2-2) indicate a multi-group representation of the delayed neutron emitters. This six group representation is valid for a detailed study of reactor dynamics, however, in the scope of this study the reactor is considered as a power source which is investigated from the terminal point of view. Therefore, without loss of generality, and for the convenience and simplicity (i.e., equipment limitation in the analog system), the kinetic equations are represented by a one group delayed neutron approximation. Equations (2-1) and (2-2) thus become:

$$\dot{n} = \frac{\delta k}{\ell} n - \frac{\beta n}{\ell} + \lambda c \tag{2-3}$$

$$\dot{c} = \frac{\beta n}{\ell} - \lambda c \tag{2-4}$$

The magnitudes of the variables n and c are far greater than the magnitudes of the other system variables. In order to facilitate the use of the analog computer, equation (2-3) and (2-4) will be normalized by dividing through by their steady state values n(0) and c(0).

$$\dot{n}/n(0) = \left(\frac{\delta k}{\ell}\right) \frac{n}{n(0)} - \left(\frac{\beta}{\ell}\right) \frac{n}{n(0)} + \frac{\lambda c}{n(0)} \tag{2-5}$$

$$\frac{\dot{c}}{c(0)} = \left(\frac{\beta}{\ell}\right) \frac{n}{c(0)} - \frac{\lambda c}{c(0)} \tag{2-6}$$

Next defining:

$$N = \frac{n}{n(0)} \; ; \; D = \frac{c}{c(0)} \; ; \text{ and } K = \frac{\delta k}{\beta}$$

noting that in steady state:

$$N = D = 1, \quad \dot{N} = \dot{D} = K = 0 \; ; \text{ and } \quad n(0) = \lambda \; c(0)$$

Equations (2-5) and (2-6) thus become:

$$\dot{N} = \frac{\beta}{\ell} (KN - N + D) \qquad (2\text{-}7)$$

$$\dot{D} = \lambda (N - D) \qquad (2\text{-}8)$$

The term K above is the effective net reactivity existing within the reactor. In steady state there is a constant neutron population and therefore K must be zero. To establish K at this zero value the reactivity attributed to rod position must be off set by the inherent internal negative feedback (internal feedback is discussed in some detail in Chapter 3). Thus the algebraic equation for K is:

$$K = K_R + \alpha_F \; T_F + \alpha_c \; T_{AV} \qquad (2\text{-}9)$$

T_F is the average fuel temperature and T_{AV} is the average coolant temperature. The coefficients α_F and α_c are coefficients of reactivity feedback. It should be noted that α_F and α_c are negative numbers.

B. HEAT TRANSFER EQUATIONS FROM FUEL AT T_F TO COOLANT AT T_{AV}

For a given change in temperature the fuel will absorb heat in the amount of

$$q_f = m_f \, c_f \, (T_2 - T_1) \qquad T_2 > T_1 \qquad (2\text{-}10)$$

Where q_f is the heat absorbed by the fuel, m_f is the mass of the fuel, c_f is the specific heat of the fuel and $T_2 - T_1$ is the change in fuel temperature. The time rate of change of the heat absorbed in this manner is:

$$\dot{q}_f = \frac{2}{2t} \, m_f \, c_f \, (T_2 - T_1) \qquad (2\text{-}11)$$

Consider one reactor fuel element of Figure (2-1). The temperature profile of the fuel varies along the axial length Z and depends upon:

1) the heat input from the reactor.

2) the coolant flow rate, which is constant.

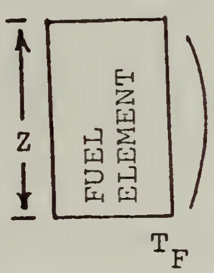


Fig. (2-1). TEMPERATURE PROFILE OF SINGLE FUEL ELEMENT.

The first of several assumptions must now be made. Assume that the temperature profile does not change shape or form

during transient conditions and that in the small signal analysis c_f and m_f are constant. Then at any axial position Z the rate of heat absorption by the fuel is dependent upon the time derivative of fuel temperature. Further considering changes from steady state where T_1 is a constant, it then follows:

$$\dot{q}_f = m_f \ c_f \ \dot{T}_F \tag{2-12}$$

The general heat transfer equation is [12]

$$\dot{q} = k_1 \ \nabla^2 \ T \tag{2-13}$$

axial (z direction) flow can be neglected in comparison to radial flow. For a homogeneous right cylinder fuel element equation (2-13) becomes:

$$\dot{q} = \frac{k_1}{r} \frac{2}{2r} \left(r \frac{2T}{2r} \right) \tag{2-14}$$

employing the power balance for each fuel element where Power = time rate of change of heat

$$\begin{pmatrix} \text{Power generated} \\ \text{by the fission} \\ \text{process} \end{pmatrix} = \begin{pmatrix} \text{Power} \\ \text{absorbed} \\ \text{by the fuel} \end{pmatrix} + \begin{pmatrix} \text{Power} \\ \text{transferred} \\ \text{to the coolant} \end{pmatrix}$$

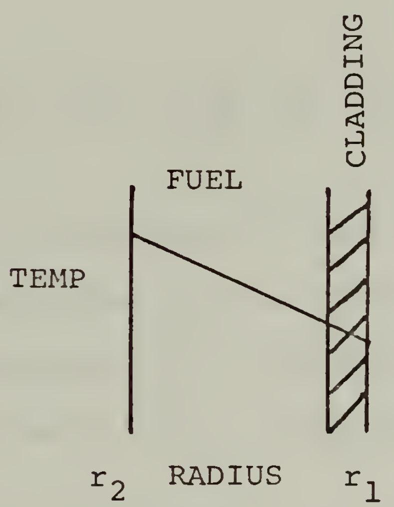
Considering the case where k_1 is independent of space variables, the (differential) power balance equation across a differential radius r is:

$$P = m_f \ c_f \ \dot{T}_F + k_1 \quad \frac{2^2 T}{2t^2} + \frac{1}{r} \frac{2T}{2r} \tag{2-15}$$

or rearranging

$$\dot{T}_f = \frac{P}{m_f\ c_f} - \frac{k_1}{m_f\ c_f}\left(\frac{2^2T}{2t^2} + \frac{1}{r}\,\frac{2T}{2r}\right) \tag{2-16}$$

Since c_f and k_1 are both functions of temperature, the above equation is a non-linear partial differential equation. Following the assumption that the temperature profile does not shape or form and that 2T/2r is a constant as shown in Figure (2-2)



$$\frac{2T}{2r} = \frac{\Delta T}{\Delta r} = \frac{T_F - T_{AV}}{r_2 - r_1}$$

$$\frac{2^2T}{2r^2} = 0$$

$$TAV = \frac{T_H + T_c}{2}$$

Fig. (2-2). TEMPERATURE PROFILE ACROSS FUEL ELEMENT AND CLADDING.

thus equation (2-16) becomes

$$\dot{T}_F = \frac{P}{m_f\ c_f} - \frac{k_1}{m_f\ c_f}\, r\left(\frac{T_F - T_{AV}}{r_2 - r_1}\right) \tag{2-17}$$

The power generated is a function of the neutron population [1]. Thus:

$$\frac{P}{m_f\ c_f} = f(N) = A_1\ N$$

The term $\frac{k_1}{m_f\ c_f\ r(r_2 - r_1)}$ has the units of $\sec^{-1}$ and can be related to a time constant τ_1. Therefore equation (2-17) becomes:

$$\dot{T}_f = A_1\ N - \frac{1}{\tau_1}\ (T_F - T_{AV}) \tag{2-18}$$

C. HEAT BALANCE FOR COOLANT IN THE CORE

Under the same assumptions as stated above the rate of heat absorbed by the coolant as it passes through the fuel elements is:

$$\dot{q}_c = m_c\ c_c\ \frac{d}{dt}\ (T_H - T_C) \tag{2-19}$$

Where $\dot{q}_c$ is the rate of heat transferred to the coolant, m_c is the mass flow rate of the coolant, c_c is the specific heat of the coolant, T_H is the hot leg temperature and T_C is the cold leg temperature. Employing a power balance as above

$$\begin{pmatrix}\text{Power transferred}\\ \text{to the coolant}\end{pmatrix} = \begin{pmatrix}\text{Power absorbed}\\ \text{by the coolant}\end{pmatrix} + \begin{pmatrix}\text{Power transferred}\\ \text{to the boiler}\end{pmatrix}$$

$$\frac{k_1}{r}\left(\frac{T_F - T_{AV}}{r_2 - r_1}\right) = m_c\ c_c\ \dot{T}_{AV} + \frac{m_c\ c_c}{\tau_o}\ (T_H - T_C) \tag{2-20}$$

Where τ_o is the time for a unit volume of coolant to flow through the reactor.

$$\dot{T}_{AV} = \frac{k_1\ (T_{AV} - T_F)}{r(r_2 - r_1)\ m_c\ c_c} - \frac{1}{\tau_o}\ (T_H - T_C) \tag{2-21}$$

again $\frac{k_1}{r(r_2 - r_1) m_c c_c}$ can be related to a time constant τ_2.

Thus:

$$\dot{T}_{AV} = \frac{1}{\tau_2} (T_{AV} - T_F) - \frac{1}{\tau_o} (T_H - T_C) \qquad (2\text{-}22)$$

Rearranging

$$\dot{T}_{AV} = \frac{T_F}{\tau_2} - \frac{T_{AV}}{\tau_2'} + \frac{2T_C}{\tau_o} \qquad (2\text{-}23)$$

where

$$\tau_2' = \frac{\tau_o \tau_2}{\tau_o + 2\tau_2}$$

D. HEAT TRANSPORT DELAYS

There are time delays associated with the transfer of heat from one portion of the plant to another. In developing the model for the time delays the following assumptions must be made:

1) there is no heat loss in the piping between the reactor and the boiler.
2) there is no mixing of fluids in the system piping.

1. <u>Time Delay From Reactor to Boiler</u>

The boiler inlet temperature, T_{BI}, is a delayed version of the reactor outlet temperature T_H.

$$T_{BI} = T_H (t - \tau_3) \qquad (2\text{-}24)$$

Where T_{BI} is the boiler inlet temperature and τ_3 is the transport time between reactor and boiler. Rearraning

$$T_H = T_{BI}\ (t + \tau_3) \tag{2-25}$$

Expanding in a Taylor Series

$$T_H = T_{BI} + \tau_3\ \dot{T}_{BI} + \ldots \tag{2-26}$$

Solving for $\dot{T}_{BI}$

$$\dot{T}_{BI} = \frac{1}{\tau_3}\ (T_{BI} - 2\ T_{AV} + T_C) \tag{2-27}$$

2. <u>Time Delay From Boiler to Reactor</u>

Following the above reasoning

$$T_C = \frac{1}{\tau_4}\ (T_C - 2\ T_B + T_{BI}) \tag{2-28}$$

Where T_B is the average temperature in the boiler.

E. HEAT BALANCE FOR HEAT EXCHANGER (BOILER)

The development of the model of a counter flow heat exchanger in which there is a change of phase (water to steam) in one flow path is a difficult task. It is not the intention of this chapter to present a rigourous derivation of the equations used in the system model, but rather it is to give insight into the physical relationships from which these equations were obtained. Therefore a more heuristic argument will be used in the development of the steam generator model.

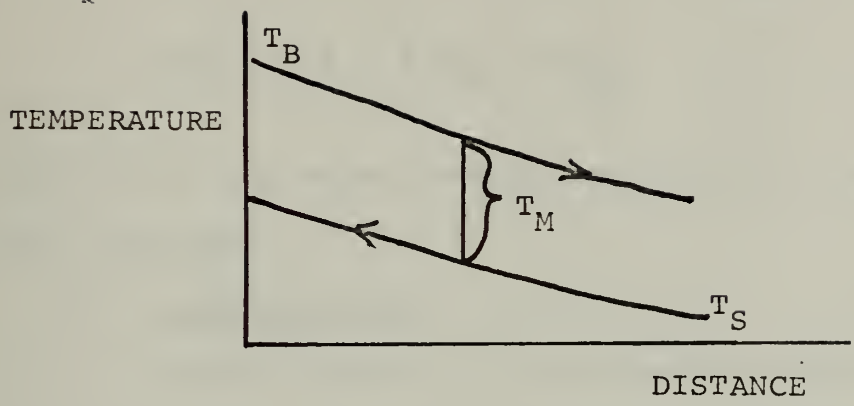


Fig. (2-3). TEMPERATURE PROFILE FOR COUNTER FLOW HEAT EXCHANGER.

There are several additional assumptions which must now be made:

1) There is no heat lost or gained with the surroundings.
2) The specific heat of the individual fluids remains constant.
3) The overall coefficient of heat transfer (k_2) is uniform through the heat exchanges.
4) From Figure (2-3) the mean temperature difference throughout the heat exchanges $T_m = T_B - T_S$

1. Primary-Side

Consider a small section of heat exchanger having a transfer area dA. Then the incremental heat transfer is:

$$dQ = k_2 \, dA \, \Delta T_n = c_c \, m_c \, dT_c \tag{2-29}$$

integrating yields

$$Q = k_2 \, A[T_B - T_S] = c_c \, m_c \, [T_{BI} - T_{BO}] \tag{2-30}$$

Since k_2, A, c_c, and m_c are all constant, equation (2-30) reduces to

$$2(T_B - T_S) = E(T_{BI} - T_{BO}) \tag{2-31}$$

Where T_S is the steam temperature and A is the area of the heat exchanger.

2. Secondary-Side

Energy balance in the secondary-side consist of the amount of power transferred to the secondary, $k_1A(T_B - T_S)$, equal to the power absorbed within the boiler plus the power which is transferred to the turbine. To bring this relation into a linear, ordinary differential equation some additional assumptions are required. These are:

1) The thermal capacity of the boiler metal, steam and coolant can be lumped.
2) The power demand on the boiler can be measured as a fraction of normal load. ψ will represent this condition with $\psi = 1$ representing the normal load.
3) The heat exchanged in the boiler occurs at a single point within the heat exchanger.

Therefore the flow of heat delivered to the turbine may be represented as the product of the throttle opening, represented by K_T, and the steam pressure. Since the steam pressure is upon the steam temperature, the term $\frac{2P_S}{2T_S}$ T_S can be used to represent steam pressure. Thus the heat balance for the secondary side becomes:

$$k_1A(T_B - T_S) = (m_c\ c_c + m_s\ c_s)\ \dot{T}_S + K_T\ \psi\ \frac{2P_S}{2T_S}\ T_S \tag{2-32}$$

Where m_s and c_s are the mass flow rate of the steam and the specific heat of the steam respectfully. Rearranging

$$\dot{T}_S = \frac{k_1 A}{m_c\ c_c + m_s\ c_s} [T_B - T_S] + \frac{K_T\ \psi}{(m_c\ c_c + m_s\ c_s)} \quad \frac{2P_S}{2T_S} T_S \tag{2-33}$$

Which reduces to

$$\dot{T}_S = - \frac{1}{\tau_5} \left(-T_B + T_S + K_T\ \psi\ \frac{2P_S}{2T_S}\right) \tag{2-34}$$

F. FINAL MODEL WITH SPECIFIC SYSTEM PARAMETERS

The development of the specific equations for the system is carried out in detail in Appendix A, they are summarized here for convenience.

1. Reactor Kinetics

$$N = 6.4\ (KN-N+D) \tag{2-35}$$

$$D = \ .1\ (N-D) \tag{3-36}$$

$$K = K_R - 1.57 \times 10^{-2}\ T_F - 3.14 \times 10^{-2}\ T_{AV} \tag{3-37}$$

where $N(0)=1$, $D(0)=1$ and $K_R(0)=24.7275$.

2. Heat Transfer From Fuel to Coolant

$$\dot{T}_F = - \frac{1}{2}\ (T_F - T_{AV}) + 30.0\ N \tag{2-38}$$

where $T_F(0)=565°F$.

3. Heat Balance for Coolant

$$\dot{T}_{AV} = 10\ T_F - 50\ T_{AV} + 40\ T_C \tag{2-39}$$

where $T_{AV}(0)=505°F$.

4. Time Delays

$$\dot{T}_{BI} = - \frac{1}{2} (T_{BI} - 2 T_{AV} + T_c) \tag{2-40}$$

where $T_{BI}(0)=520°F$.

$$\dot{T}_c = - \frac{1}{2} (T_c - T_{BI}/3 - \frac{2}{3} T_S) \tag{2-41}$$

where $T_c(0)=490°F$.

5. Heat Balance for Steam Generator Secondary-Side

$$\dot{T}_S = - \frac{1}{5} (- \frac{2}{3} T_{BI} + (0.667 + 0.0665 \psi) T_S) \tag{2-42}$$

where $T_S(0)=565°F$.

G. METHOD OF SOLUTION

Because of the multiplication of the time dependent variables K and N in equation (2-7) and because K is a function of N the entire set of dependent equations which describe the plant dynamics is non-linear. Since (in general) non-linear differential equations do not lend themselves readily to analytical solutions, computer simulation will be used to solve these equations.

III. INHERENT REACTOR CHARACTERISTICS

There are several important characteristics of nuclear power plants which greatly effect the stability and controllability of the system. The two most significant factors relating to the system performance are the existance of delayed neutron emmiters and the negative internal reactivity feedback associated with temperature changes within the system. Both of these phenomena are discussed below in some detail.

A. PROMPT AND DELAYED NEUTRONS

At the instant of fission there are neutrons (prompt) which are produced instantaneously. Fortunately, for control purposes, a small portion (β) of all neutrons produced during the fission process are delayed in time [1, 2]. These delayed neutrons are the result of the decay of fission products. As noted previously these delayed neutrons are grouped in six distinct groups with different delay constants (λ). Analysis of the effects which delayed neutrons exhibit on the reactor kinetics can best be shown by the evaluation of the kinetics equations (2-3) and (2-4) with and without the delayed neutrons considered.

The analysis of the prompt and delayed neutron response which follows is a study of the reactor operating at a power level low enough so that feedback effects (discussed later in this chapter) are negligible. An analagous situation in

a power plant would be in the start-up power range of the reactor.

In this evaluation the input to the reactor is in the form of a perturbation to the steady state reactivity k. The input was limited to a maximum value (β) of (0.0064), for when δk exceeds β the reactor is in a response condition described as "prompt critical." "Prompt critical" refers to the fact that the nuclear fission chain reaction can be maintained by means of prompt neutrons alone. If this condition occurs neutron density, and hence power increase rapidly from the instant of input, making the reactor difficult to control and may cause what is referred to as a start up accident. The "prompt critcial" operation is avoided in practice.

1. Kinetic Equations Assuming No Delayed Neutrons

Under the assumption of no delayed neutrons, equations (2-3) and (2-4), which describe the reactor kinetics in steady state with a one group delayed neutron approximation, reduce to the single equation:

$$\dot{n} = \frac{\delta k n}{\ell} \tag{3-1}$$

Solving for n

$$n = n(0)\, e^{\frac{\delta k}{\ell} t} \tag{3-2}$$

Dividing by n(0), substituting the system parameters found in Appendix A, and assuming $\delta k = 0.003$ equation (3-2) now becomes:

$$N = e^{3t} \tag{3-3}$$

2. Reactor Kinetic Equations with a One Group Delayed Neutron Emitter

Under the one group delayed emitter assumption, equations (2-3) and (2-4) are unchanged. With the substitution of system parameters from Appendix A and from the solution of these equations carried out in Ref. [11], the solution of these equations for N is:

$$N = 1.88\ e^{+0.0884\ t} - 0.88\ e^{-3.4\ t} \qquad (3\text{-}4)$$

3. Comparison of Results

The effect of the delayed neutrons is readily seen by the comparison of equations (3-3) and (3-4). With no delayed neutrons the reactor neutron population has increased by a factor of 8,100 in 3 seconds, however when delayed neutrons are present the reactor neutron population has only increased by a factor of 2.5 in 3 seconds. Plots of N versus time for both of these situations are shown in Figures (3-1) and (3-2). As time passes (i.e., time greater than 10 seconds) the first term of equation (3-4) becomes dominant and the response will follow a positive exponential increase. However, in the time span of interest (a small time after the disturbance) the affect of the delayed neutrons is to greatly reduce the rate of change of neutron flux. It is the natural occurance of these delayed neutrons which makes it possible to operate a nuclear reactor with the degree of safety required. Even as small a fraction as 0.64 percent of the total neutron population acting as delayed neutrons has the effect of making the entire reactor control problem greatly simplified [2, 3].

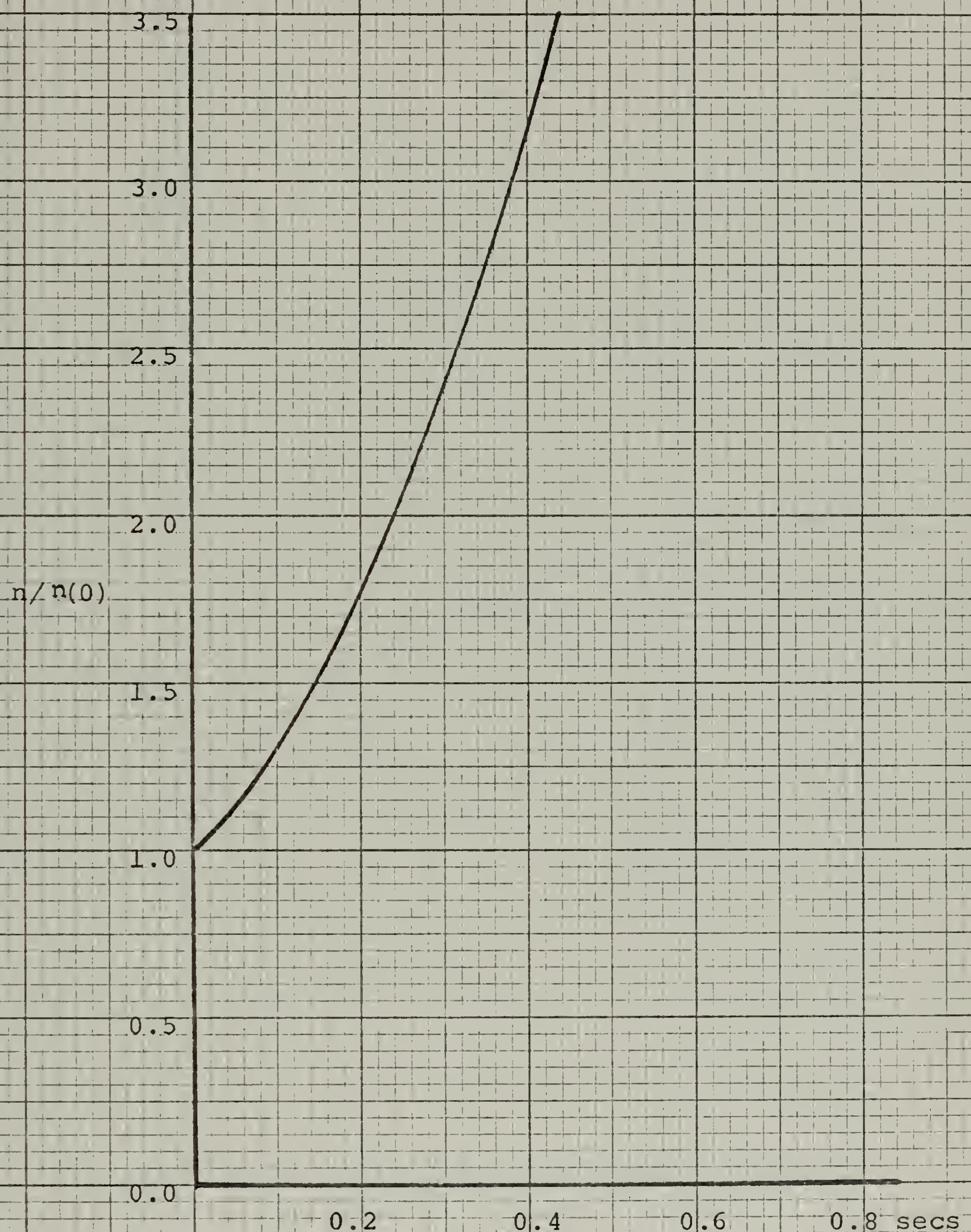


Fig. (3-1). TIME RESPONSE OF REACTOR CONSIDERING ONLY PROMPT NEUTRONS $\delta k = 0.003$.

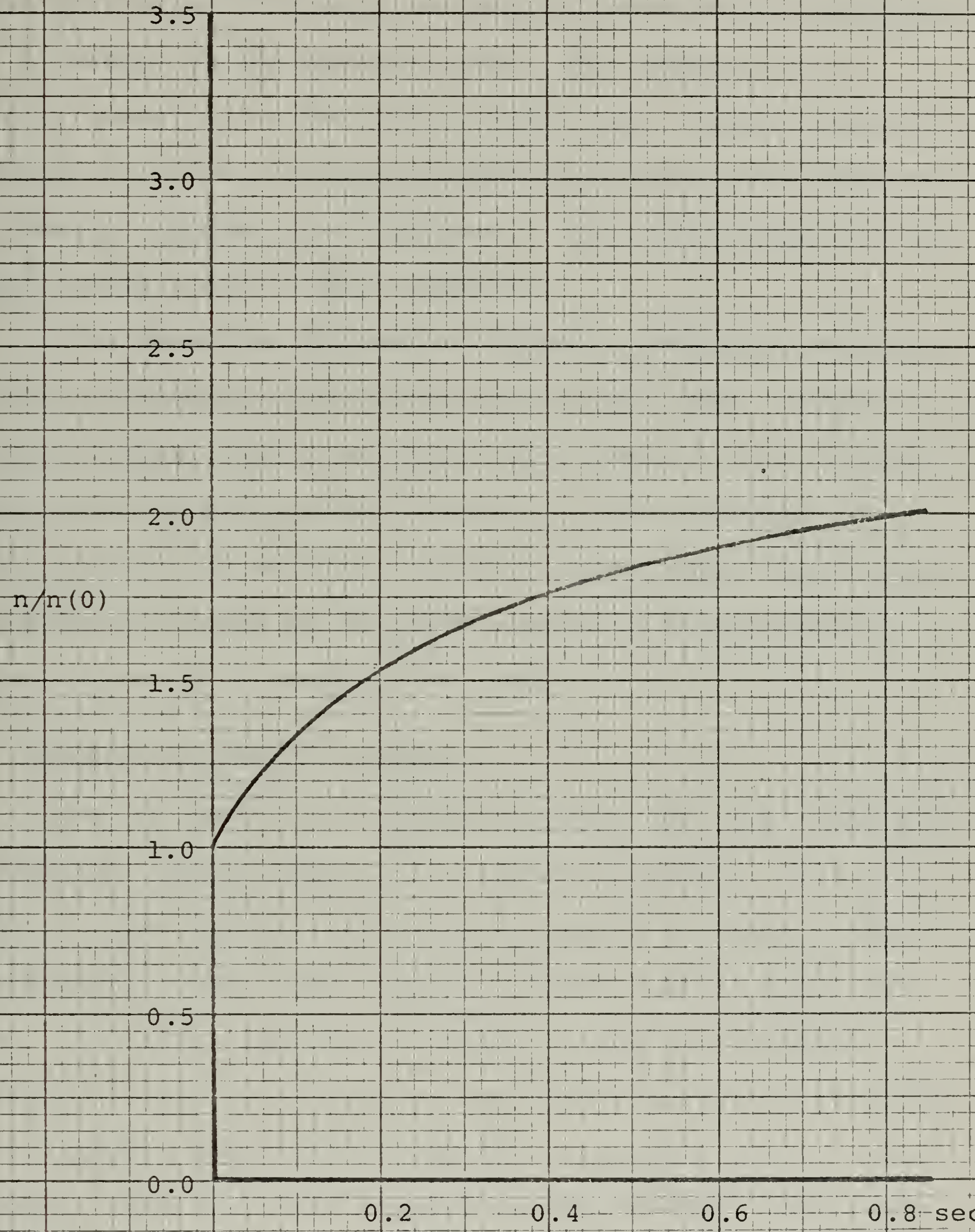


Fig. (3-2). TIME RESPONSE OF REACTOR WITH PROMPT AND DELAYED NEUTRONS $\delta k = -0.003$.

It is considered pertinent to re-emphasize that the above discussion of prompt and delayed neutrons was conducted for a reactor in the startup range. The response of a reactor operating in the normal power range (where reactivity feedback is strong) is significantly different than that of the reactor operating at low power. This difference will be demonstrated later in this chapter.

It should be pointed out however, that the delayed neutron phenomenon also has an undesirable facet. While it is readily obvious that it restricts the growth of the neutron population for positive δk inputs, it also restrains the reduction rate of neutron population which would result from a negative δk input as the power demand is reduced during normal operation or during shut down.

B. INTERNAL REACTIVITY FEEDBACK

In the operation of a nuclear reactor the power generated is directly proportional to the neutron population [1]. In order for the neutron population to remain constant at some steady state power level, equation (2-7) must be equal to zero. For this situation to occur, K, the net internal reactivity which is the sum of both the reactivity attributed to the control rod position and the reactivity due to internal feedback, must be zero. Changes in the feedback reactivity depend upon temperature variations and fission-product--poisoning which are discussed below

To insure the stability of the reactor the overall coefficient of reactivity feedback must be negative [13].

1. Temperature Effects

When there is a power change in a nuclear reactor there is considerable additional energy released. Since it is not possible to transfer all of the liberated energy to the load, there are temperature fluctuations throughout the reactor and associated coolant.

Temperature variations affect reactivity in two distant ways: first, the mean energy of the thermal neutrons and hence their nuclear cross-section vary with temperature; and, second, the mean free path length and leakage probabilities are functions of temperature [2, 3]. These effects result in a net negative reactivity. A reactor power plant of the type described above, having a negative temperature coefficient of reactivity will tend to be self-controlling in response to load changes and limited external reactivity variations [14].

In hetrogeneous reactors (the type under study here) the fuel and the coolant are at different temperatures and at different physical states. Therefore, there will be specific effects associated with both the fuel and coolant [15].

Temperature effects upon reactivity manifest themselves in temperature coefficients of reactivity. (α_F and α_C).

2. Fission-Product Poisoning

When a reactor is operating at power certain fission products, notably Xenon 135 and Samarium 149, are accumulated. These products have large nuclear cross sections and absorb the excess neutrons, which are vital in maintaining the chain reaction. The rate of formation of these products is

dependent upon the rate of fission occuring at the specific power level. Eventually the rate of formation and the loss of the absorbing nuclei become equal and equilibrium exists. However, when the reactor is shut down, these substances, which result mainly from the decay of other fission products, continue to grow and may reach their highest concentration several hours after shut down occurs. Since the production of these fission products continues after shut down, it is entirely possible that there may be enough negative reactivity remaining to restrict the power build up of the reactor until these products decay to a level where the inherent positive reactivity in the control rods can overcome their adverse reactivity effects.

The above fission product phenomenon does not essentially affect the performance of a reactor operating at power. For this reason these effects are not considered in the scope of this study.

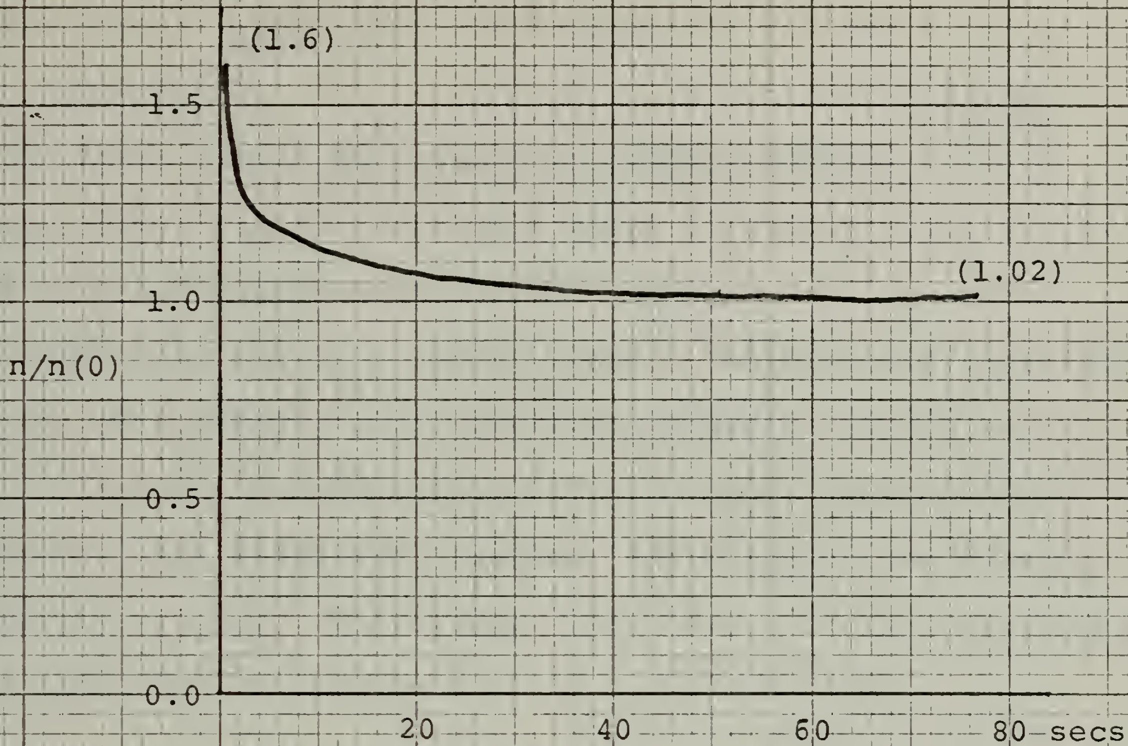
3. Net Internal Reactivity

The net reactivity in dollar units, due to both rod position and to temperature can be expressed as:

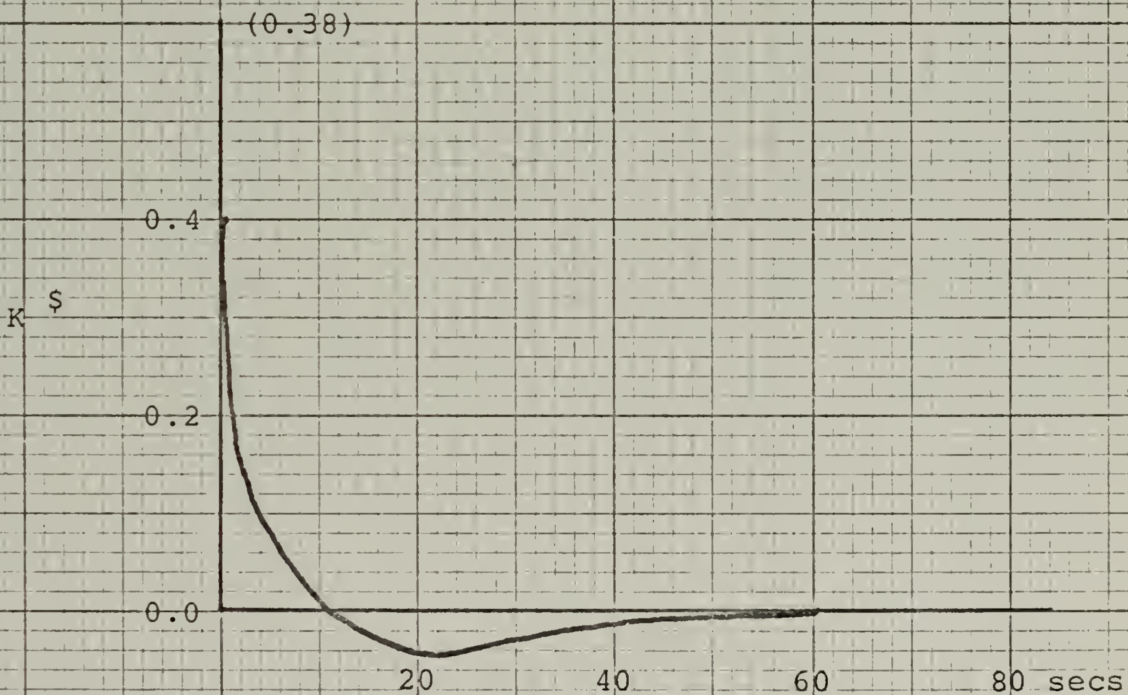
$$K = K_R + \alpha_f T_F + \alpha_c T_{AV}$$

C. TOTAL EFFECT OF DELAYED NEUTRONS AND NEGATIVE REACTIVITY FEEDBACK

The total effect of the delayed neutrons and negative reactivity can best be illustrated in Figure (3-3). These plots are for the specific plant under study. The power plant



NEUTRON DENSITY VERSUS TIME



NET REACTIVITY VERSUS TIME

Fig. (3-3)

disturbance is a step input of magnitude K = 0.469 dollars (i.e., $\delta k = 0.003$).

The input causes the reactor's neutron density to rise sharply from a steady state unit value to a value of 1.6. If there were no delayed neutrons this neutron density would rise faster and obtain a larger value. The negative reactivity feedback acts rapidly and limits the excursion of the net reactivity to a maximum value of 0.38. After approximately 60 seconds, the reactivity feedback has adjusted the internal reactivity to zero. The steady state effect of the disturbance is to raise the neutron density to a level of 1.02.

IV. DIGITAL AND ANALOG SIMULATION OF POWER PLANT WITHOUT EXTERNAL CONTROL

A. DIGITAL SIMULATION

The system equations were programmed and solved on the IBM 360-67, using the Naval Postgraduate School Library Program (DRKGS) for the integration. This integration method utilizes a forth-order "Runge-Kutta" solution with double precision arithmetic.

Time response of the system to load changes of +40 percent are shown in Appendix B and are tabulated for all experimental load changes at the end of this chapter.

1. Load Changes

The system was subjected to standardized step load changes disturbances of $\pm$ 20 percent and $\pm$ 40 percent. These load changes were selected because a 20 percent load change represents a change which a maritime power plant should be able to respond to quickly, and a 40 percent load change represents a feasible change which could be encountered during an accidental situation.

2. Results

The results of the simulation of this hypothetical system were compared against the time response of marine power plant designs of similar size [6, 8]. It was found that the simulation used in this study compared well with the referenced systems. The steady state (after the load distrubance) values

of the system variables coincided well with their counterparts. The simulated system demonstrated a slightly more oscillatory behavior than the actual and proposed systems referenced. This oscillatory behavior is attributed to the simplified system model used in this study. Even with the small oscillatory differences, the simulation represents the actual system with a sufficient degree of accuracy that it is quite suitable for an initial study of marine reactor power plants.

3. Discussion of Results

a) The power plant, without benefit of an external control system, was shown to be stable to all subjected load changes. The system displayed a second order well damped response to load changes.

b) Temperature changes, including overshoots, are not severe and should not cause thermal stress problems.

c) System response, although sluggish, is fast enough for use as a maritime power plant.

B. ANALOG SOLUTION

The analog simulation of the system was carried out on the Comcor CI-5000 computer. This computer has a $\pm$ 100 volt dynamic range. The CI-5000 computer is interfaced with the XDS-9300 digital computer. This hybrid pair will be used later in the control section of this study.

1. Prompt-jump Approximation

Figure (B-1) of Appendix B shows that the time rate of change of N is not large. For ease of analog simulation (i.e., to reduce the number of high gain amplifiers required

in the simulation) equation (2-7) can be modified as follows:

$$\frac{\dot{N}}{6.4} = KN - N + D \qquad (4\text{-}1)$$

making the approximation that $\dot{N} = 0$ and solving for N

$$N = \frac{D}{1-K} \qquad (4\text{-}2)$$

This approximation is called the prompt-jump approximation since any change in K is reflected instantaneously in N. The system equations using this assumption were simulated on the IBM 360-67 and it was found that this approximation does not introduce errors greater than 2 percent in the system response.

2. <u>System Variables to Computer Variables</u>

The system variables such as temperature and relative neutron population must be related to analog computer variables which are voltages. The system equations and initial conditions in terms of computer variables are developed in Appendix C. These equations are summarized below for convenience:

a. Reactor Kinetics

$$\bar{N} = \frac{5\ \bar{D}}{5-\bar{K}} \qquad (4\text{-}3)$$

$$\dot{\bar{D}} = 0.1\ (\bar{N}-\bar{D}) \qquad (4\text{-}4)$$

$$\bar{K} = 3.5\ \bar{K}_R - 0.471\ \bar{T}_F - 0.942\ \bar{T}_{AV} \qquad (4\text{-}5)$$

where

$\bar{N}(0) \;=\; 10$ volts

$\bar{D}(0) \;=\; 10$ volts

$\bar{K}_R(0) \;=\; 35.36$ volts

b. Heat Transfer From Fuel to Coolant

$$\dot{\bar{T}}_F = -\,0.5\,[\bar{T}_F - \bar{T}_{AV}] + 0.5\,\bar{N} \qquad (4\text{-}6)$$

Where

$\bar{T}_F(0) = 94.17$ volts

c. Heat Balance for Coolant

$$\dot{\bar{T}}_{AV} = -50\,\bar{T}_{AV} + 10\,\bar{T}_F + 40\,\bar{T}_c \qquad (4\text{-}7)$$

Where

$$T_{AV}(0) = 84.17 \text{ volts} \qquad (4\text{-}8)$$

d. Time Delays

$$\dot{\bar{T}}_{BI} = -0.5\,[\bar{T}_{BI} - 2\,\bar{T}_{AV} + \bar{T}_c] \qquad (4\text{-}9)$$

$$\dot{\bar{T}}_c = 0.5\,[-\bar{T}_c + \tfrac{2}{3}\,\bar{T}_S + \tfrac{1}{3}\,\bar{T}_{BI}] \qquad (4\text{-}10)$$

Where

$\bar{T}_{BI}(0) \;=\; 86.67$ volts

$\bar{T}_c(0) \;=\; 81.67$ volts

e. Heat Balance for Steam Generator Secondary-Side

$$\dot{\bar{T}}_S = [0.133\ \bar{T}_{BI} - 0.1324\ \bar{T}_S - 0.0027\ \bar{\psi}\ \bar{T}_S] \quad (4\text{-}11)$$

Where

$$\bar{T}_S(0) = 79.17 \text{ volts}$$

The analog computer diagram for the solution of these equations is shown in Figure (4-1).

3. Comparison of Analog and Digital Solution

Below is shown two tables comparing the results of the digital and analog computer solutions. Because of the inherent resolution quality characteristics of the digital computer, the digital solution will be used as the base for determining errors. Settling time is defined as the amount of time required for the magnitude of K(t), the net internal reactivity, to return to a value of $\pm$ 0.005 after the applied step load disturbances.

From the tabulated results it is apparent that the analog and digital solutions are in such close agreement that the analog solution is adaquate for the remainder of this study. Henceforth the digital computer is used as an auxiliary computational tool to solve specific problems but is not used in the evaluation of the overall simulation.

The value of the system variables obtained from the analog simulation were obtained through Analog to Digital conversion and printed out from the XDS-9300 digital computer.

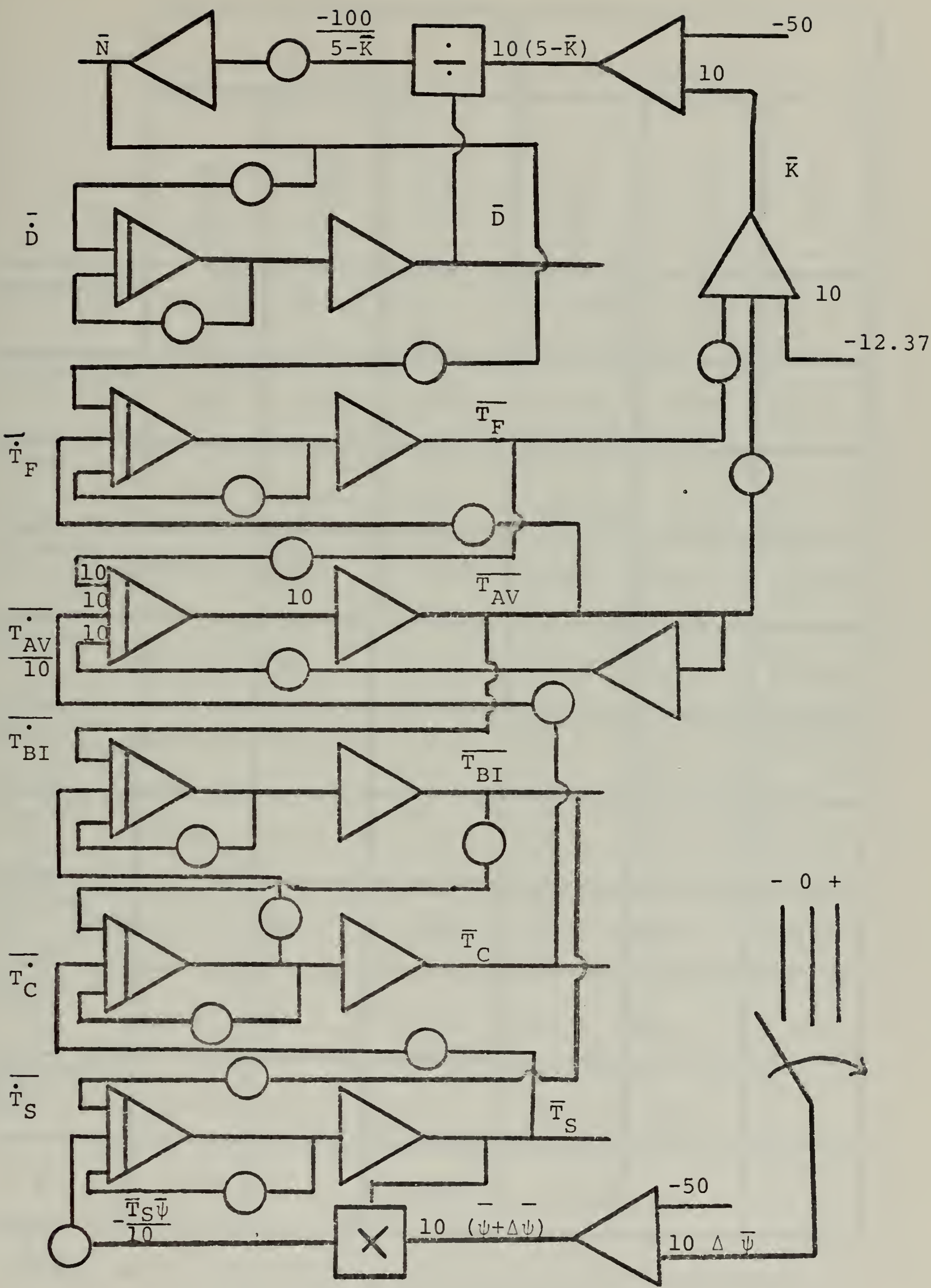


Fig. (4-1). ANALOG COMPUTER DIAGRAM FOR BASIC POWER PLANT.

	$\Delta\ \psi = +0.2$			$\Delta\ \psi = -0.2$		
	ANALOG	DIGITAL	% ERROR	ANALOG	DIGITAL	% ERROR
K (MAX/MIN) \$	0.068	0.066	3	-0.081	-0.084	3.58
N (MAX/MIN)	1.21	1.20	0.83	0.81	0.79	2.53
N (SS)	1.21	1.19	1.68	0.82	0.80	2.5
T_F (MAX/MIN) °F	573.0	572.7	0.052	557.1	556.8	0.054
T_F (SS) °F	573.0	572.6	0.07	557.4	557.1	0.054
T_{AV} (MAX/MIN) °F	499.8	500.5	0.14	509.2	510.1	0.178
T_{AV} (SS) °F	500.4	501.2	0.16	508.2	509.0	0.156
T_S (MAX/MIN) °F	465.3	464.9	0.086	486.2	486.0	0.042
T_S (SS) °F	466.0	465.5	0.107	485.2	484.9	0.0625
SETTLING TIME	51.5	47.4	8.6	49.5	52.0	5
ψ	1.199	1.2	0.083	0.801	0.8	0.013

TABLE (4-1). COMPARISON OF ANALOG AND DIGITAL SOLUTION FOR A STEP LOAD CHANGE OF 20 PERCENT.

	$\Delta\ \psi = +0.4$			$\Delta\ \psi = -0.4$		
	ANALOG*	DIGITAL	% ERROR	ANALOG*	DIGITAL	% ERROR
K(MAX/MIN) \$	0.117	0.119	1.68	-0.188	- 0.198	5.0
N(MAX/MIN)	1.40	1.38	1.45	0.586	0.563	4.6
N(SS)	1.38	1.37	0.73	0.618	0.595	4.0
T_F(MAX/MIN) °F	580.1	580.1	0.0	548.6	548.0	0.123
T_F(SS) °F	580.1	579.9	0.034	549.4	548.8	0.123
T_{AV}(MAX/MIN) °F	495.7	496.3	0.12	515.0	516.2	0.232
T_{AV}(SS) °F	497.0	497.5	0.10	512.2	513.0	0.156
T_S(MAX/MIN) °F	456.0	455.3	0.154	498.0	495.0	0.60
T_S(SS) °F	457.0	456.4	0.133	495.2	495.2	0.0
SETTLING TIME	54.2	50.0	8.4	108.0	98.0	10.0
ψ	1.397	1.4	0.215	0.604	0.6	0.667

TABLE (4-2). COMPARISON OF ANALOG AND DIGITAL SOLUTION FOR A STEP LOAD CHANGE OF 40 PERCENT.

V. CONTROL

The purpose of an external control scheme for a nuclear power plant must be to set up the desired steady state operating conditions, and to restrict transients in vital plant variables to be consistant with design limitations. The above purposes should be obtained without any significant reduction in the inherent stability characteristics of the plant without external control.

Examination of the basic plant transient responses in Appendix B indicates that the system under study is very stable when subjected to load changes. It is however, desirable, as pointed out below, to limit the excursion of certain plant variables from their steady state normal value. In a plant with a large degree of inherent stability direct control of plant variables may be attempted. If a plant has poor inherent stability, neutron-level control should also be used [3].

A. STEADY STATE PROGRAMMING

The pattern that the plant temperatures, pressures, and flow rates assume as a function of power-demand is oftened referred to as a program [4]. A specific control strategy can be devised to obtain the desired program. The two most significant programs for discussion are the constant T_{AV} and constant T_S programs.

1. <u>Constant T_{AV}</u>

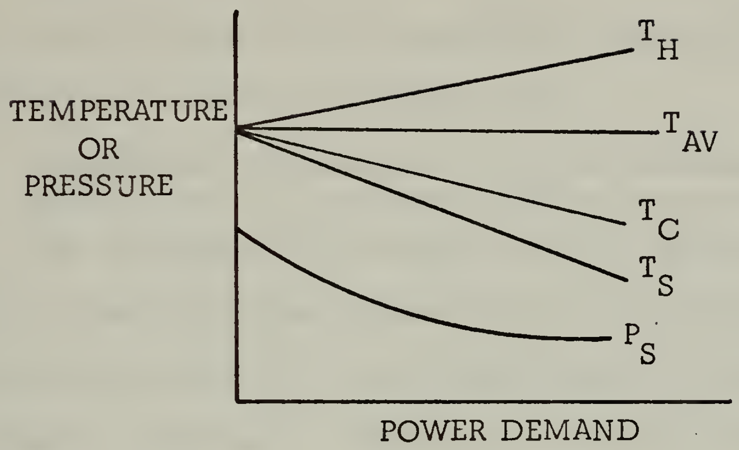


Fig. (5-1). SKETCH OF TEMPERATURE AND PRESSURE PROFILES FOR A CONSTANT T_{AV} PROGRAM.

Figure (5-1) is a graphical representation of the constant T_{AV} program. At zero power demand, all significant plant temperatures (except T_F) are at the same value. As the power demand increases, the steam temperature and hence pressure decrease, while T_H and T_C vary in such a manner as to keep T_{AV} constant.

2. <u>Constant T_S</u>

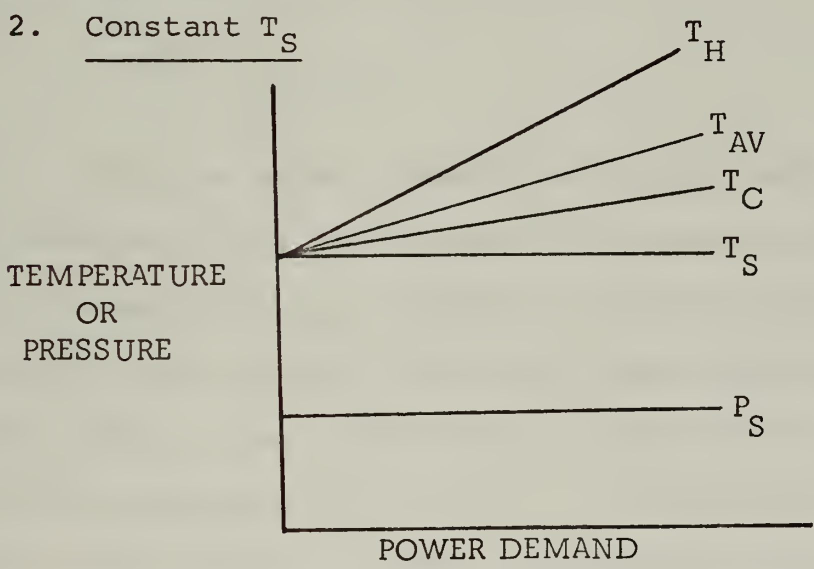


Fig. (5-2). SKETCH OF TEMPERATURE AND PRESSURE PROFILES FOR A CONSTANT T_S PROGRAM.

Again at no load all pertinent temperatures are equal. As the load increases the temperature and the pressures follow the pattern as shown in Figure (5-2).

3. <u>Comparison of Constant T_{AV} and Constant T_S Programs</u>

The constant T_{AV} program is the one preferred by the reactor. The principle advantage of the constant T_{AV} program for this type reactor is that the primary coolant pressure also remains constant. Therefore the need for an external pressurizer in the primary circuit is reduced.

One additional advantage occurs if the plant under study had been fuel with an uranium alloy vice an oxide of uranium. For a metal alloy fuel the feedback of internal reactivity associated with changes in fuel temperature is insignificant when compared to the feedback associated with changes in average coolant and therefore can be ignored [3]. For this case equation (2-9) becomes:

$$K = K_R + \alpha_c T_{AV} \qquad (5\text{-}1)$$

Since in stead stake K must equal zero, it is apparent from equation (5-1) that there is no need for control rod motion to maintain T_{AV} constant. However the plant under study is fueled with UO_2 and changes in fuel temperature have a strong affect on net reactivity. Examination of equation (2-9) now indicates that there is a requirement to change control rod position to keep the average temperature constant.

The advantages of the constant T_S Program are apparent, from Figure (5-2), in the design of the secondary loop of the

plant. A constant T_S program also provides a constant steam pressure which allows for an optimum design of the steam plant with conventional controlling devices.

The two primary disadvantages of the constant T_S program are first, allowing T_{AV} to vary over a large range (required to keep T_S constant) may cause serious problems in the primary coolant pressure system and second, when the plant is changing from one steady state power level to another, the plant must fight the tendency of the coolant negative temperature coefficient to hold T_{AV} constant.

With the exception of Nautilus, which uses a constant T_C program, all Naval plants use a constant T_{AV} program [16]. The N. S. Savannah utilizes a control plan for a constant T_{AV} program with a plus or minus 3°F dead zone about the normal load T_{AV} value [17].

B. DESIGN OF THE EXTERNAL CONTROL SYSTEM

Consistant with the advantages of the constant T_{AV} program, the simulated power plant will be controlled under a constant T_{AV} program. As noted in section (5A-3) control rod movement is required in order to obtain the desired program. The next decision to be made is at what rate may reactivity be externally introduced into the reactor. The answer to this and other design questions are discussed below.

1. Reactivity Rate

The one basic underlying consideration which dictates the design of a control system for a nuclear power plant is safety. As shown in Figure (3-3) the power plant operating

at normal power can withstand a step reactivity input of magnitude $\delta K = .469\$$. This however is not the case for a reactor operating at reduced power levels where the negative reactivity feedback effect is reduced. Therefore in order to provide a maximum safety margin to avoid startup or low power accidents input reactivity rates will be restricted to a value which can be handled by a reactor operating at lower power. The solution to equations (2-7) and (2-8) for ramp input of reactivity, K(t), does not give a valid solution to the problem because there is no feedback considered. Externally introduced reactivity rates are determined largely from empirical data on the individual plants involved. A reasonable reactivity rate for a plant of this size is 10¢/sec (0.1 $/sec) [7].

It is feasible that this rate of reactivity may be controlled by a manual means. However, it was desired that the system be fully automated therefore an automatic controller was used.

2. Automatic Controller

The automatic controller consists of a hybrid combination of an analog control rod positioning system and a digital system for comparison of T_{AV} plus the time derivative of T_{AV} with the reference value of 505°F. It was decided to use the $\pm$ 3°F dead band error detector of N. S. Savannah as a valid method to obtain a discontinuous controller. The principle advantage of a discontinuous controller is that it utilizes the reactor inherent stability characteristics to reduce wear on the control rod mechanisms. The dead zone, although it

must be consistent with the design temperature (i.e., pressure) fluctuations in the primary loop, discriminates against noise on the T_{AV} signal. A block diagram description of the entire simulation, plant plus controller, is represented in Figure (5-3). The design of the control rod positioning system and a discussion of the hybrid simulation along with transient response curves for a step load input of +40 percent of the system with external control are shown in Appendix D.

3. Evaluation of External Controller

The external controller was implemented and incorporated into the simulation. The system was subjected to step load disturbances of plus and minus 20 and 40 percent. The improvement in system performance can be seen from tables (5-1) and (5-2). The reduction of the excursions of T_{AV}, both transient and steady state, and the reduction of settling time were considered significant.

The plant was subjected to step load changes of up to 80 percent (an unrealistic change). It was found that the system was stable and that the system response improved up to step load inputs of 60 percent. Above 60 percent, the system response deteriorated and eventually went into a limit cycle when the step load disturbance became greater than 75 percent. The deteriorated performance and eventual instability of the system were attributed to the fact that the derivative of T_{AV} became very large and began to work in the opposite direction of the T_{AV} excursions rather than in normal anticipatory mode of a position plus derivative feedback controller.

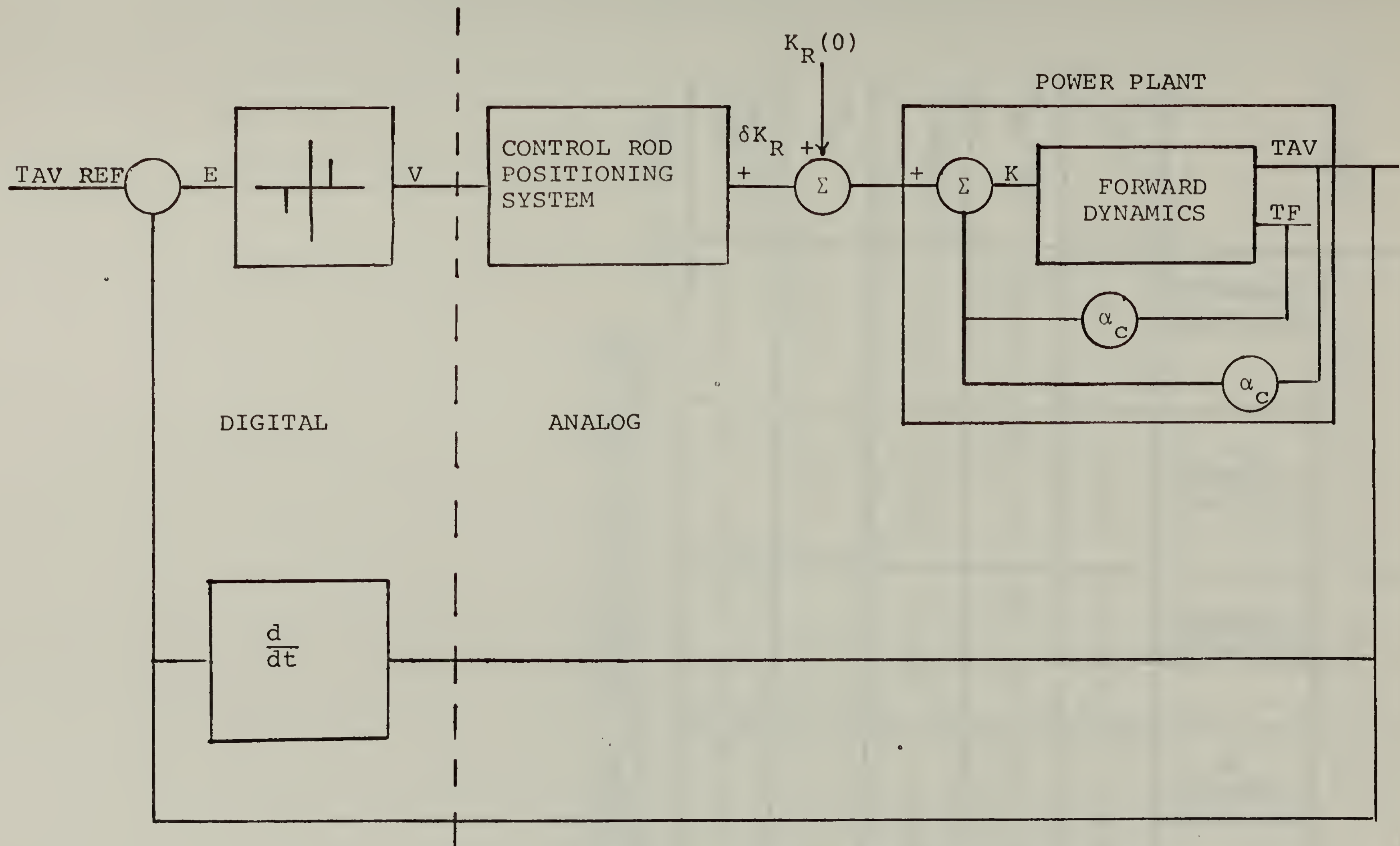


Fig. (5-3). BLOCK DIAGRAM OF POWER PLANT WITH EXTERNAL CONTROLLER.

	$\Delta\psi = +0.2$		$\Delta\psi = -0.2$	
	EXTERNAL CONTROLLER	NO EXTERNAL CONTROLLER	EXTERNAL CONTROLLER	NO EXTERNAL CONTROLLER
ΔT_{AV}	- 3 °F	- 5.2°F	+ 4.2°F	+ 4.2°F
$T_{AV}(SS)$	505.6°F	500.4°F	505 °F	508.2°F
K(MAX/MIN)	0.12$	0.068	- 0.16$	- 0.081
ΔK_R	+ 0.1$	-----	0.0	-----
SETTLING TIME	36 sec	51.5 sec	40.0 sec	49.5 sec

TABLE (5-1). COMPARISON OF SIMULATED POWER PLANT WITH AND WITHOUT EXTERNAL CONTROLLER FOR A STEP LOAD CHANGE OF 20 PERCENT.

	$\Delta\psi = +0.4$		$\Delta\psi = -0.4$	
	EXTERNAL CONTROLLER	NO EXTERNAL CONTROLLER	EXTERNAL CONTROLLER	NO EXTERNAL CONTROLLER
ΔT_{AV}	- 5.1	- 9.3°F	+ 7.2°F	+10.0°F
$T_{AV}(SS)$	508.0°F	497.0°F	507.4°F	512.2°F
K(MAX/MIN)	0.17$	0.117$	- 0.32	- 0.188$
ΔK_R	+ 0.6$	-----	+ 0.6$	-----
SETTLING TIME	36 sec	54.2 sec	77 sec	108 sec

TABLE (5-2). COMPARISON OF SIMULATED POWER PLANT WITH AND WITHOUT EXTERNAL CONTROLLER FOR A STEP LOAD CHANGE OF 40 PERCENT.

The controller was considered adequate for all reasonable power changes.

VI. CONCLUSIONS AND RECOMMENDATIONS

The following conclusions and recommendations are offered as a result of the investigation contained in this report:

A. CONCLUSIONS

1. A properly designed pressurized water nuclear reactor (i.e., one with a negative temperature coefficient of reactivity) possesses a great deal of inherent stability when the associated power plant is operating in the normal power range. It was observed that the negative temperature coefficient of reactivity served to stabilize the reactor for disturbances in both load and input reactivity.

2. The naturally occuring phenomenon of delayed neutrons greatly simplifies the control problems associated with nuclear reactors. The effect of this phenomenon is more evident at the lower power levels before the temperature reactivity feedback effect has become affective.

3. The control of a stable nuclear power plant operating in the power range under a constant T_{AV} program with constant coolant flow rate is a relatively simple task. If sufficient stability exists in the reactor then direct control of a plant parameter (T_{AV}) can be attempted.

4. If step load changes greater than 60 percent, for which the above control system is not suitable, are considered feasible a comparator could be installed in the differentiating circuit to disregard the derivative if it becomes too large.

B. RECOMMENDATIONS

Further workers in this area should consider the following facets of reactor systems.

1. The above study was concerned with the reactor power plant dynamics when excited by both load disturbances and changes in reactivity investigated separately. Another area of interest is the response of the reactor under the condition of load changes and externally induced reactivity changes (both aiding and opposing) occuring simultaneously. This situation might represent an accidental condition.

2. The T_{AV} program used to control the power plant does not consider the maximum value to T_H. Since $T_{AV} = \frac{T_H + T_C}{2}$ the value of T_H might become too high for design considerations. To institute a control program with a constant T_{AV} and also limiting T_H to some maximum value would make the requirement of a variable coolant flow rate necessary. This added requirement would necessitate a new mathematical model. Under the assumption of a constant flow rate, the time coolant spends in the reactor (τ_0) is a constant. However, in the model for a plant where a maximum value of T_H must be considered τ_0 would be a function of T_H.

3. More sophisticated control problems, than reactor control at operating conditions, exist in the nuclear power plant control field. These are the problems associated with the reactor in the startup range and problems involved in the mechanical control of the various plant components of the secondary loop. The problems in the control of the secondary

loop result from the fact that when the reactor is controlled under a constant T_{AV} program, the steam temperature and pressure vary significantly over the operating range of the plant. It is recommended that further investigators consider the control problems of the secondary loop.

APPENDIX A

SPECIFIC SYSTEM EQUATIONS

In the development of the generalized system equation, equations (2-28) and (2-34) are expressed in terms of the variable T_B. T_B is not a state variable of the system. The equation for the primary side of the heat exchanger (2-31) can be solved for T_B and this value substiuted into the equations (2-28) and (2-34) to place them in a more suitable form. This yields:

$$\dot{T}_C = - \frac{1}{\tau_4} \left(T_C - \frac{2T_S}{e+1} - \frac{E-1}{E+1} T_{BI} \right)$$

$$T_S = -\frac{1}{\tau_5} \left(- \frac{E}{E+1} \quad T_{BI} + \left(\frac{E}{E+1} + K_T \psi \frac{2P_S}{2T_S} \right) T_S \right)$$

The system parameters for the hypothetical plant are stated below. It should be emphasized that these parameters do not represent any actual real system but that the parameters used here are obtained from various sources in the literature [5, 6, 7, 8, 9] and represent typical values which might be associated with a plant the size of the power plant of N. S. Savannah.

1) Fraction of delayed neutrons $\beta = 0.0064$

2) Average delay constant of the 6 group delayed neutrons $\lambda = 0.1$

3) Mean neutron life time $\ell = 10^{-3}$ sec

4) Steady state temperature

   A) Average fuel temperature $T_F(0) = 565°F$

   B) Average coolant temperature $T_{AV}(0) = 505°F$

   C) Boiler inlet temperature $T_{BI}(0) = 520°F$

   D) Steam temperature $T_S(0) = 475°F$

   E) Cold leg temperature $T_C(0) = 490°F$

5) Temperature coefficient of reactivity for the fuel (UO_2)

$$\alpha_F = -1.57 \times 10^{-2} \ \$/°F$$

6) Temperature coefficient of reactivity for the coolant (pressurized water)

$$\alpha_c = -3.14 \times 10^{-2} \ \$/°F$$

7) Time constants for the various heat transfer equations

   A) $\tau_1 = 2.0$ sec.

   B) $\tau_2 = 0.1$ sec.

   C) $\tau_3 = 2.0$ sec.*

   D) $\tau_4 = 2.0$ sec.*

   E) $\tau_5 = 5.0$ sec.*

8) In the region of interest the steam pressure-steam temperature relation is approximately constant

$$\frac{2P_S}{2T_S} = 5 \text{ psi/°F}$$

9) The values of $K_R(0)$, A_1, K_T, τ_0, τ_2 were determined by setting the equation in which these terms appear to

zero (steady state) and solving for the unknown variables. The results are:

A) $K_R(0) = 24.7275$ $

B) $A_1 = 30.0$

C) $E = 2.0$

D) $K_T = 1.33 \times 10^{-2}$

E) $\tau_0 = 0.05$ sec.

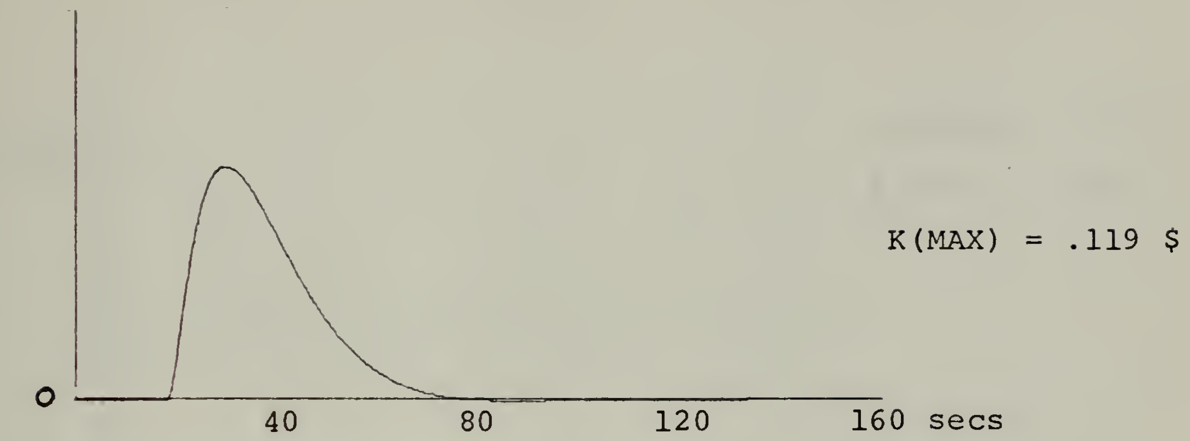
F) $\tau_2 = 0.02$ sec

*τ_3, τ_4, and τ_5 were obtained by engineering approximation and are substantially less than those mentioned for a stationary power plant [9].

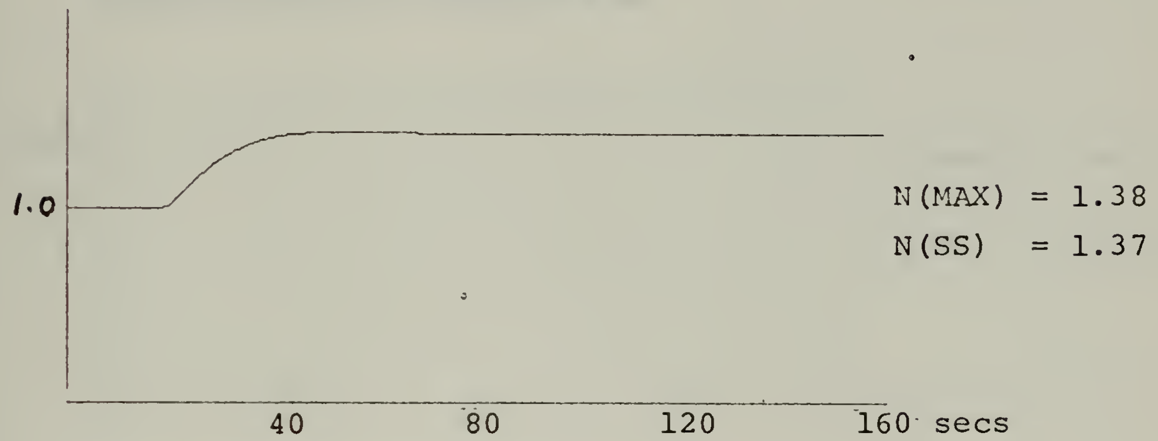
APPENDIX B

TIME RESPONSE OF POWER PLANT

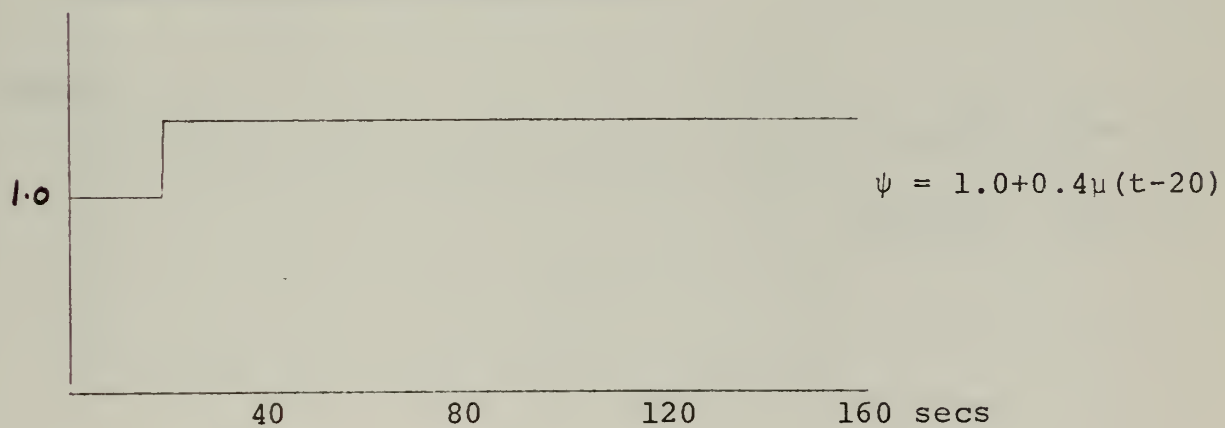
As a representative example of system time response Figures (B-1), (B-1a), (B-2) and (B-2a) of this appendix show the time response of selected system variables to a step load change of +40 percent normal load. Figures (B-1) and (B-1a) are solutions using the digital simulation. Figures (B-2) and (B-2a) are solutions utilizing the analog simulation. Pertinent values of the variables are shown adjacent to their time response. The system response for other experimental load changes are similar in nature to those shown.



NET REACTIVITY VERSUS TIME

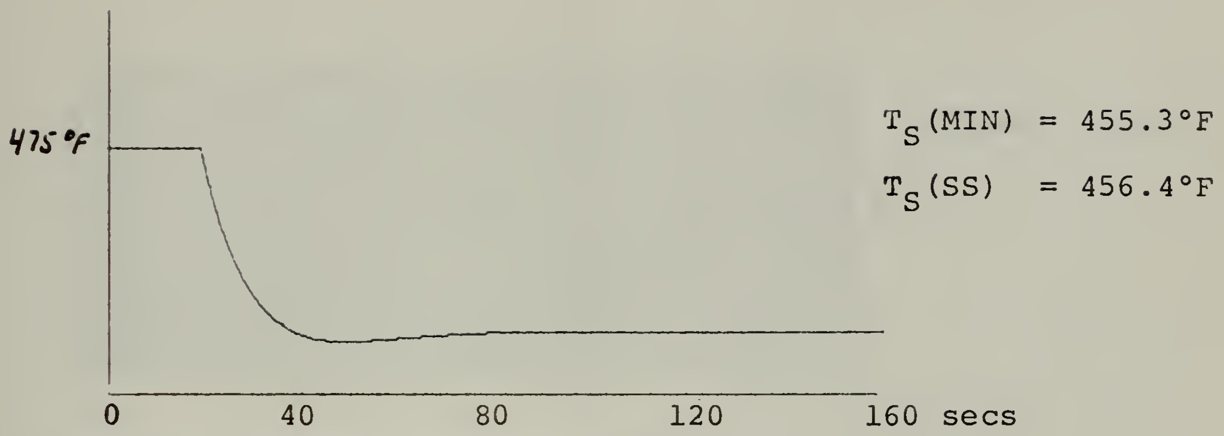


NEUTRON POPULATION VERSUS TIME

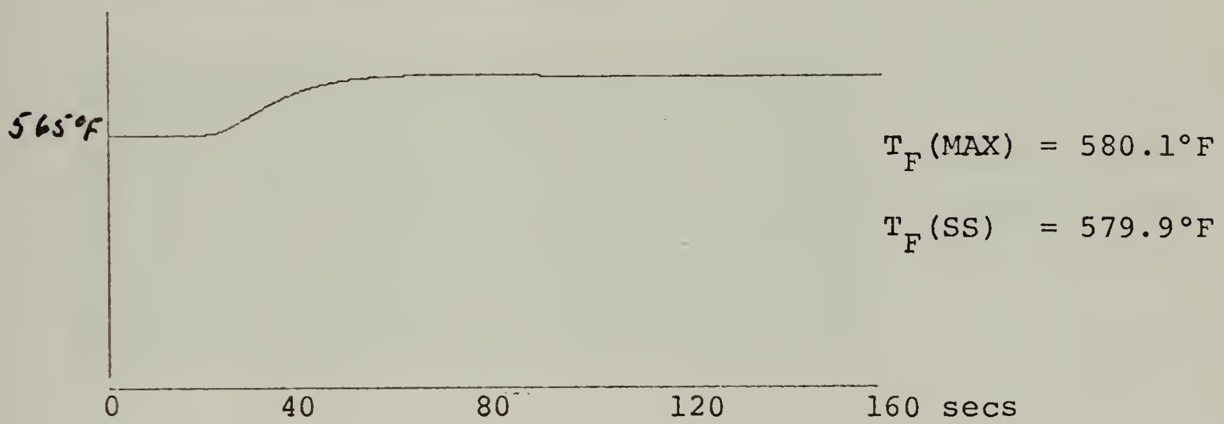


LOAD CHANGE

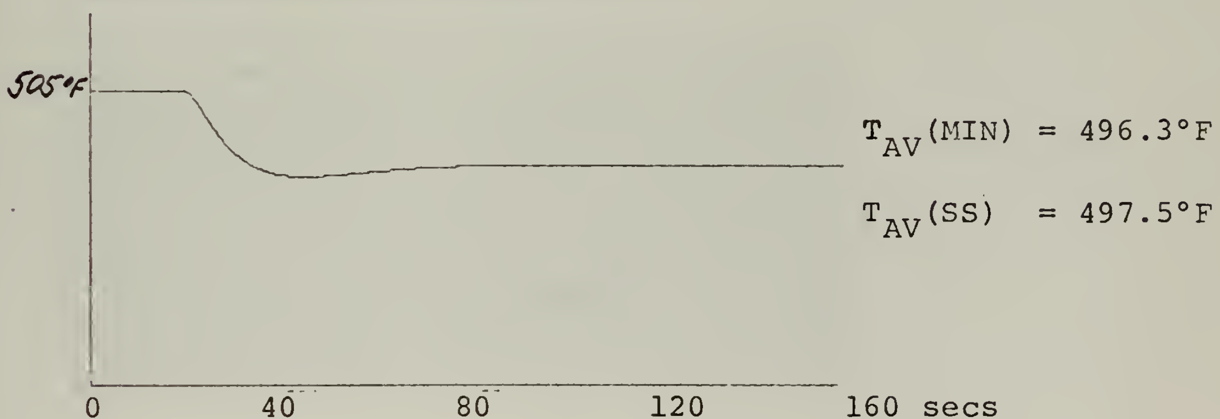
Fig. (B-1). TIME RESPONSE OF SIMULATED POWER PLANT TO A 40 PERCENT STEP LOAD CHANGE (DIGITAL).



STEAM TEMPERATURE VERSUS TIME

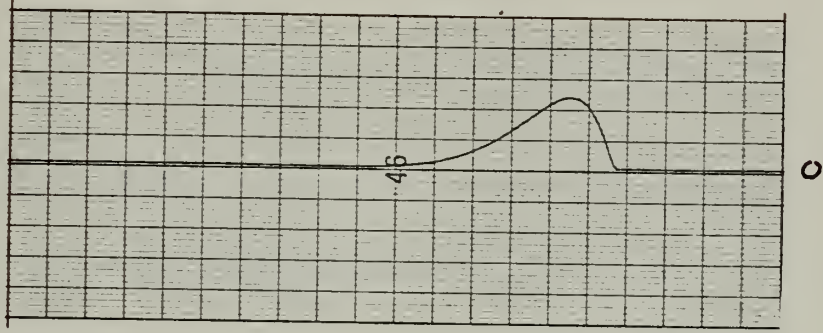


FUEL TEMPERATURE VERSUS TIME



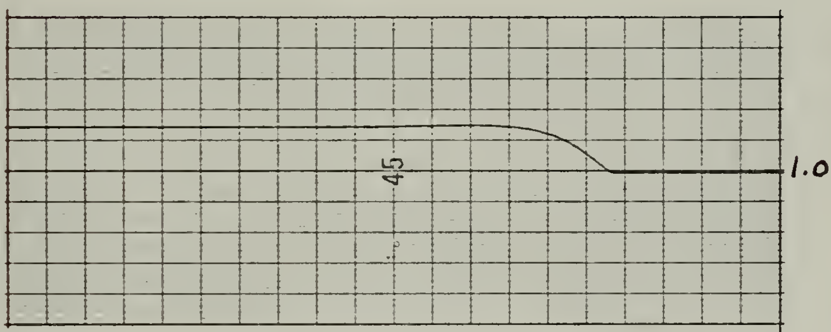
AVERAGE COOLANT TEMPERATURE VERSUS TIME

Fig. (B-1a). TIME RESPONSE OF SIMULATED POWER PLANT TO A 40 PERCENT STEP LOAD CHANGE (DIGITAL).



K(MAX) = .117 $

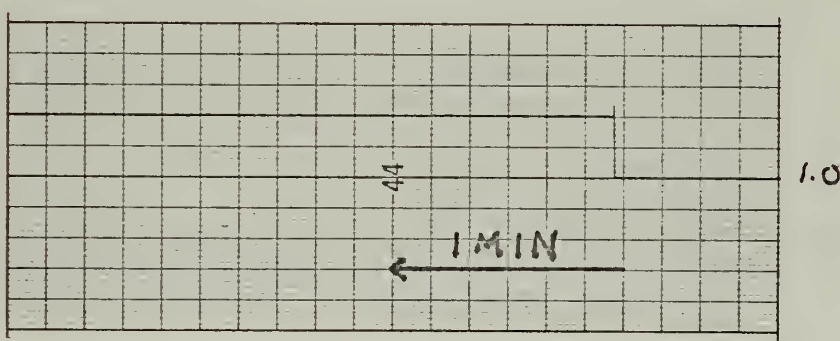
NET REACTIVITY VERSUS TIME



N(MAX) = 1.40

N(SS) = 1.38

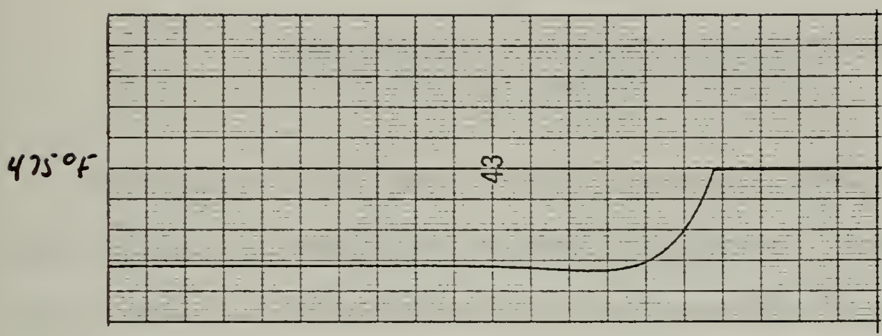
NEUTRON POPULATION VERSUS TIME



$\Delta\ \psi = +0.4$

LOAD CHANGE

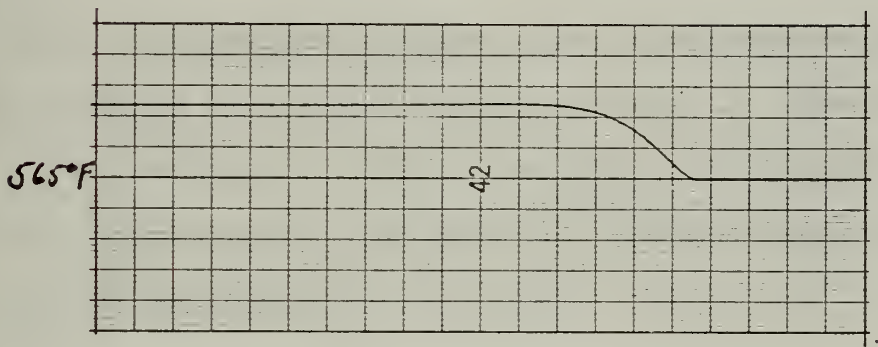
Fig. (B-2). TIME RESPONSE OF SIMULATED POWER PLANT TO A 40 PERCENT STEP LOAD CHANGE (ANALOG).



T_S(MIN) = 456.0°F

T_S(SS) = 457.0°F

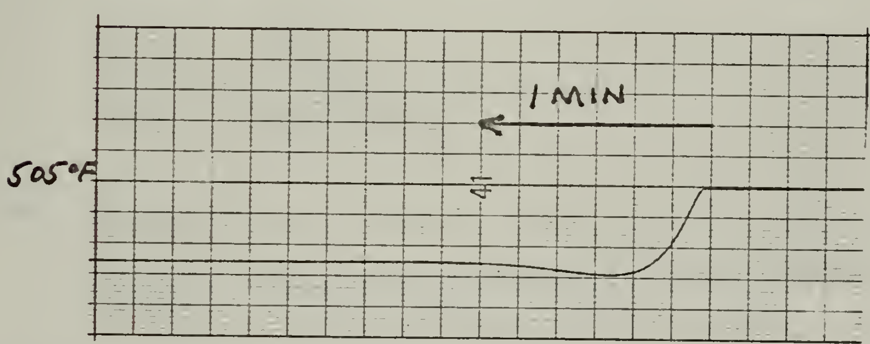
STEAM TEMPERATURE VERSUS TIME



T_F(MAX) = 580.4°F

T_F(SS) = 580.1°F

FUEL TEMPERATURE VERSUS TIME



T_{AV}(MIN) = 495.7°F

T_{AV}(SS) = 497.0°F

AVERAGE COOLANT TEMPERATURE VERSUS TIME

Fig. (B-2a). TIME RESPONSE OF SIMULATED POWER PLANT TO A 40 PERCENT STEP LOAD CHANGE (ANALOG).

APPENDIX C

PREPARATION OF SYSTEM EQUATIONS FOR USE ON ANALOG COMPUTER

Equations (4-2) and (2-36) through (2-42) describe the power plant in terms of system variables. These system variables have units such as degrees Fahrenheit and dollar units of reactivity which must be related to analog computer variables which are voltages. In addition to the conversion of units, the analog voltages have a maximum value of 100 volts so scaling of the variables may also be required.

To accomplish this conversion a scaling factor coefficient x is introduced. The relation between computer variables and system variables then becomes:

$$N = x_N \bar{N}$$

$$D = x_D \bar{D}$$

$$T_F = x_{TF} \bar{T}_F$$

and so forth.

The individual x's are equal to the maximum expected value of the system variable divided by the maximum desired voltage for the computer variables. For example:

$$T_F = x_{TF} \bar{T}_F,$$

assuming T_F (MAX) = 600°F and $\overline{T_F}$ (MAX) = 100 volts

$$\therefore \quad x_{TF} = \frac{T_F}{\overline{T_F}} = 6°F/\text{volt}$$

A summary of the scaling coefficients is as follows:

$$x_N = .1 \text{ volt}^{-1}$$

$$x_D = .1 \text{ volt}^{-1}$$

$$x_K = .2\$/\text{volt}$$

$$x_{KR} = .7\ \$/\text{volt}$$

$$x\,\psi = .1 \text{ volt}^{-1}$$

$$x_{TF} = x_{TS} = x_{TAV} = x_{TC} = x_{TBI} = 6°F/\text{volt}$$

The system equations are now altered by replacing the system variables with their corresponding computer variables and conversion factors. The initial condition of the differential equations are determined as follows:

$$\overline{T}_F(0) = \frac{T_F(0)}{x_{TF}}$$

In a final form the equations now become:

1. Reactor Kinetics

$$\bar{N} = \frac{5\bar{D}}{5-\bar{K}}$$

$$\dot{\bar{D}} = 0.1\ (\bar{N}-\bar{D})$$

$$\bar{K} = 3.5\ \bar{K}_R - 0.471\ \overline{T_F} - 0.942\ \overline{T_{AV}}$$

Where

$\bar{N}(0) = 10$ volts

$\bar{D}(0) = 10$ volts

$\bar{K}_R(0) = 35.36$ volts

2. Heat Transfer from Fuel to Coolant

$$\overline{\dot{T}_F} = -0.5\ (\bar{T}_F - \bar{T}_{AV})\ 5.0\ \bar{N}$$

Where $\bar{T}_F(0) = 94.17$ volts

3. Heat Balance for Coolant

$$\overline{T_{AV}} = -50\ \overline{T_{AV}} + \overline{T_F} + 40\ \overline{T_C}$$

Where $\overline{T_{AV}(0)} = 84.17$

4. Time Delays

$$\overline{\dot{T}_{BI}} = 0.5\ (-\overline{T_{BI}} + 2\overline{T_{AV}} - \overline{T_C})$$

$$\overline{\dot{T}_C} = 0.5\ (-\overline{T_C} + 2/3\ \overline{T_S} + 1/3\ \overline{T_{BI}})$$

Where $\overline{T_{BI}}(0) = 86.67$ volts

$\overline{T_C}(0) = 81.67$ volts

5. Heat Balance for Steam Generator Secondary-Side

$$T_S = (0.133\ \overline{T_{BI}} - .1324\ \overline{T_S} - 0.0027\ \overline{T_S})$$

Where $\overline{T_S}(0) = 79.17$ volts

$\bar{\psi}(0) = 5$ volts

APPENDIX D

CONTROLLER SIMULATION

Analytical design of the external controller could have been approached in one of two ways. First a small signal linearization of the plant dynamics could have been attempted. This method was not valid since two of the plant variables, N and D, change in almost direct proportion to load changes. For example a 40 percent load change causes a 40 percent change in N and D. This change in magnitude of the variables was outside the range of small signal application. Second a describing function could have been obtained utilizing sinusodial inputs [18]. The describing function technique is a laborious method of obtaining insight into the absolute stability limits of the system but gives little insight into the controller design problem. In this initial study the controller design and evaluation was carried out through simulation. If a more detailed study had been undertaken, a describing function would have been necessary to verify the simulation results.

The simulation of the external controller was accomplished by use of a trial and error technique utilizing the hybrid computing feature of the XDS-9300 and Comcor CI-5000 computers.

The analog voltage representing T_{AV} was sampled at a frequency of 10 samples per second (this sampling frequency is well in excess of that required by the sampling theorem). The

values of T_{AV} were stored in an array. When the above array had accumulated 10 members, an approximation to the time derivation of $T_{AV}(10) - T_{AV}(1)$ (divided by unit time). The inclusion of the derivative feedback will give the system an anticipatory response. After the subtraction the array was updated so that a new derivative was available every 0.1 second. Due to the fact that T_{AV} was changing at a rate of less than 0.5 degrees per second at distrubances below 60 percent, the approximation of the derivative of T_{AV} was considered accurate. The sum of T_{AV} and $\dot{T}_{AV}$ were subtracted from $\dot{T}_{AV}$ references (505°F). If the error were more than 3°F an error signal of $\pm$ 1 volt was sent to the analog computer.

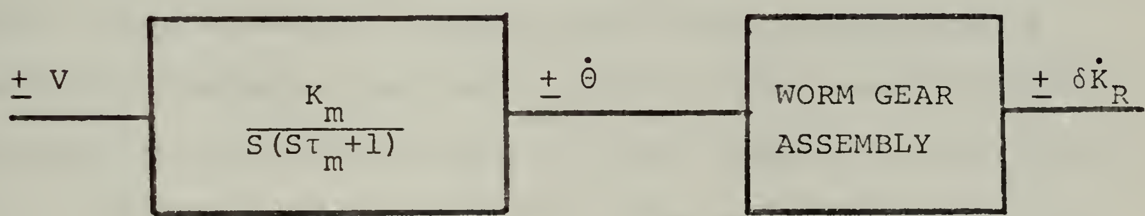


Fig. (D-1). BLOCK DIAGRAM OF CONTROL ROD POSITIONING SYSTEM.

The $\pm$ 1 volt error signal was placed into a standard electrical motor with transfer function $\frac{K_m}{S(S\tau_n + 1)}$. As indicated in Figure (D-1) in the actual plant this motor would

drive at constant speed through a worm gear assembly and remove or insert the control rods at a constant speed. The gear assembly represents only a linear gain and can be combined with the motor gain to give a new transfer function $\frac{K_{1m}}{S(S\tau_m + 1)}$. The control rod effectiveness (reactivity per inch) is a non-linear function of position and is different for individual reactors. It is assumed here that the rods are operating in a linear mode where δK_R = .1 \$/sec. Under the assumption that τ_m = .1 sec. the final transfer function becomes $\frac{1}{S(S + 10)}$. Simulation of the drive motor yields the transient responses of Figures (D-2) and (D-3). From the simulation of the control rod positioning device the rise time is 0.25 seconds and the steady state δK_R is 10.15¢/sec.

The rise time could be shortened and the steady state $\dot{K}_R$ remain constant if the forward gain were increased and a tachometer feedback loop used. However this would allow the possibility of an excessive δK_R if the tachometer loop were to fail. Since the rise time of the uncompensated system was quite satisfactory for this application the tachometer feedback loop was not employed.

Transient response of the system with external controller is shown in Figure (D-4) and (D-4a).

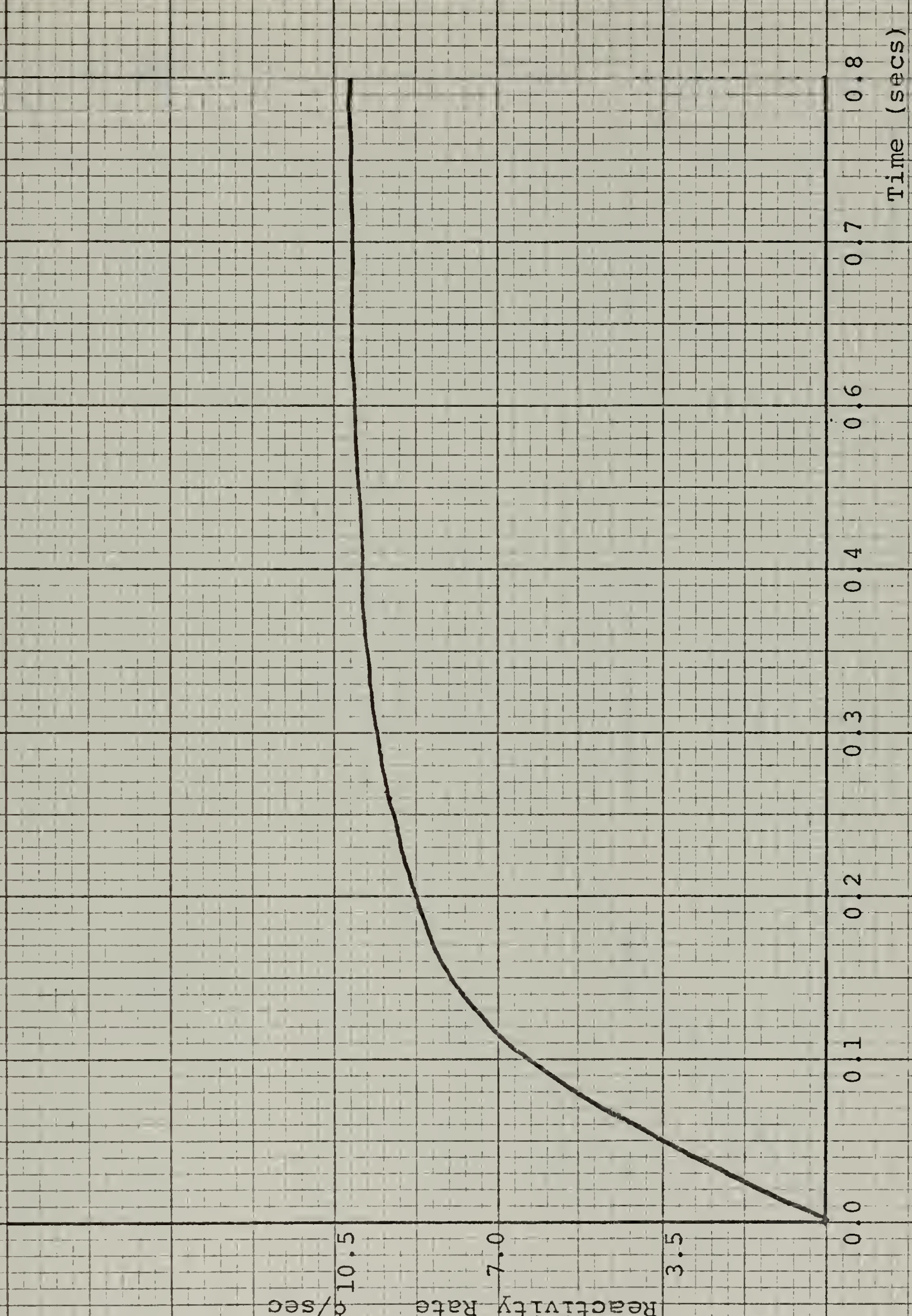


Fig. (D-2). TIME RESPONSE OF CONTROL ROD POSITIONING SYSTEM.

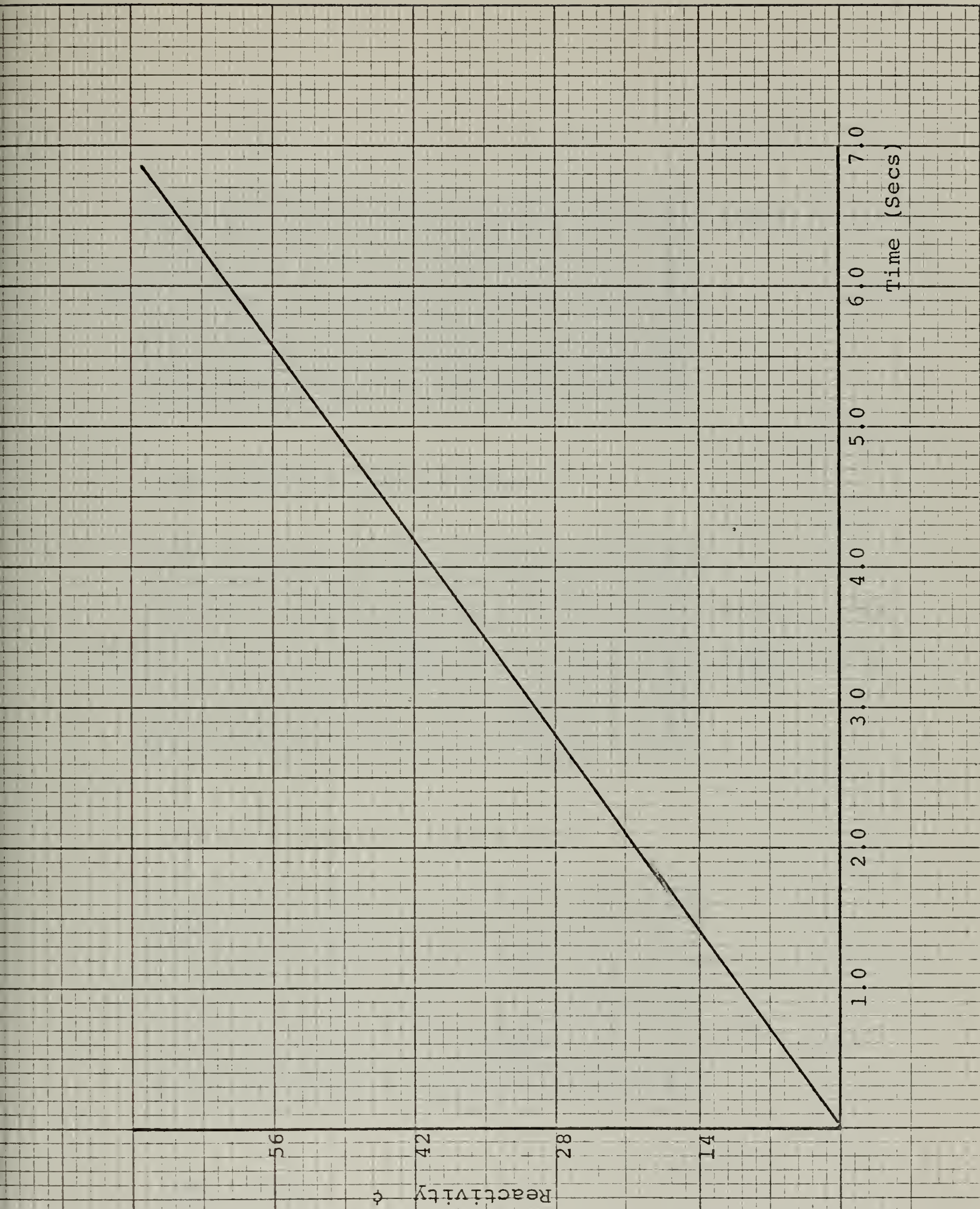
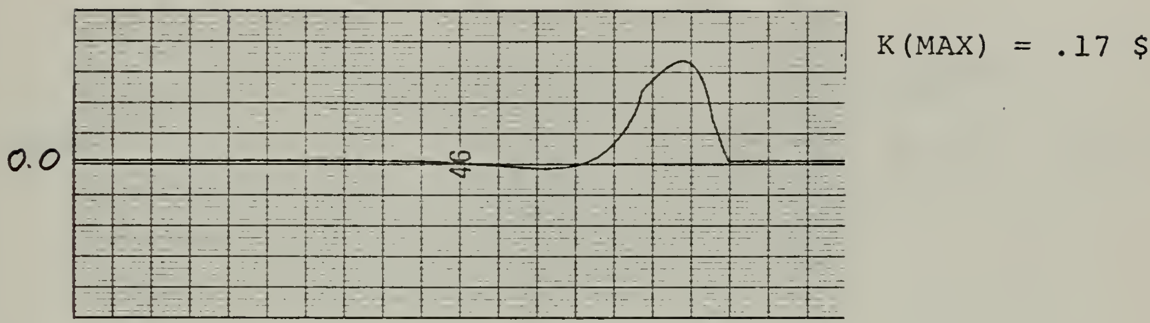
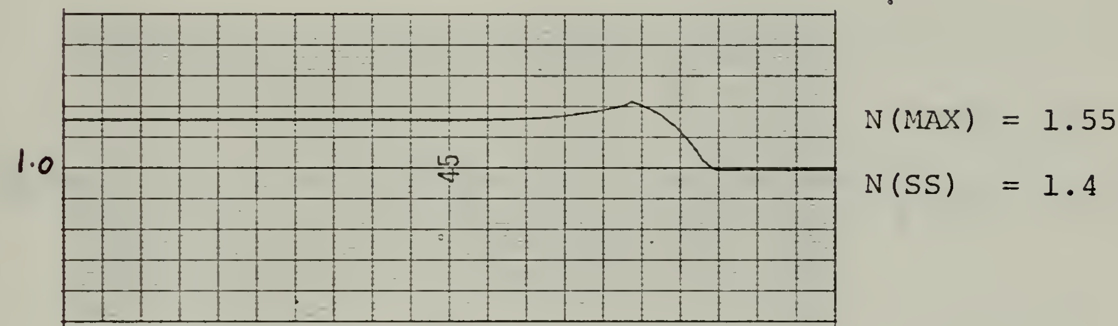


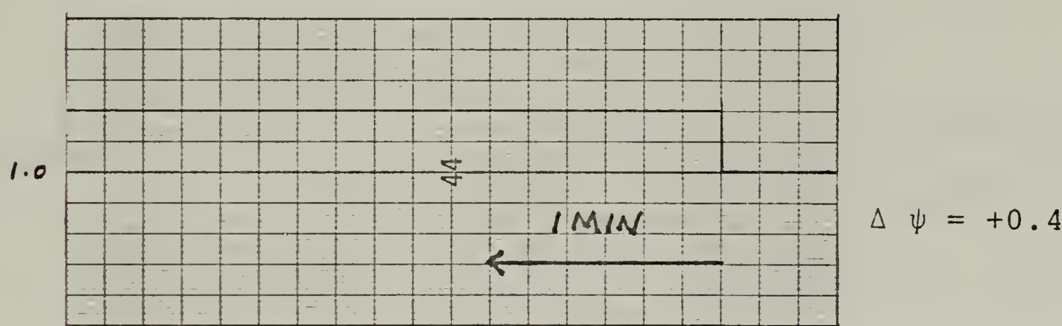
Fig. (D-3). TIME RESPONSE OF CONTROL
ROD POSITIONING SYSTEM.



NET REACTIVITY VERSUS TIME

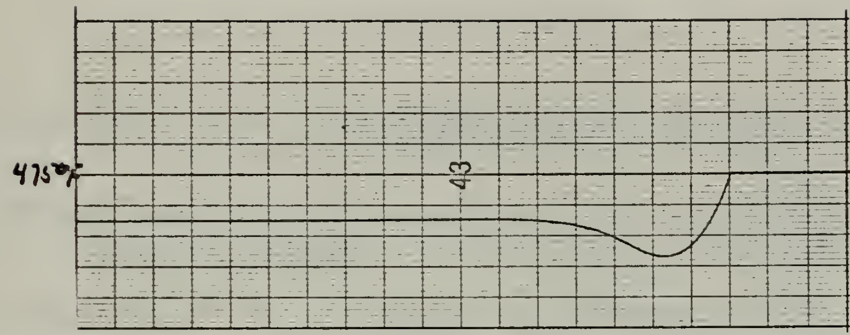


NEUTRON POPULATION VERSUS TIME



LOAD CHANGE

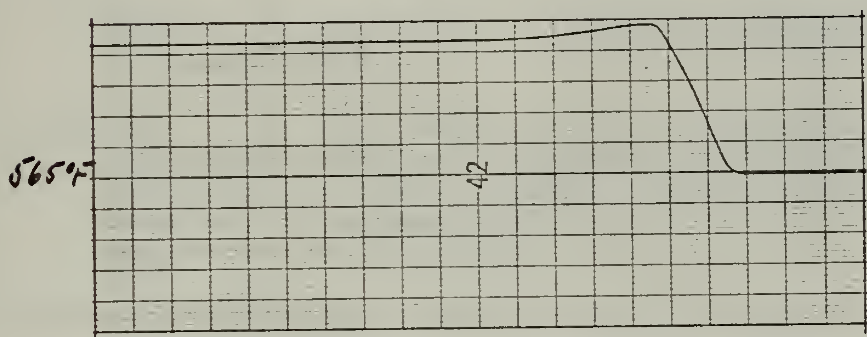
Fig. (D-4). TIME RESPONSE OF POWER PLANT WITH EXTERNAL CONTROLLER.



T_S(MIN) = 459.4°F

T_S(SS) = 465.4°F

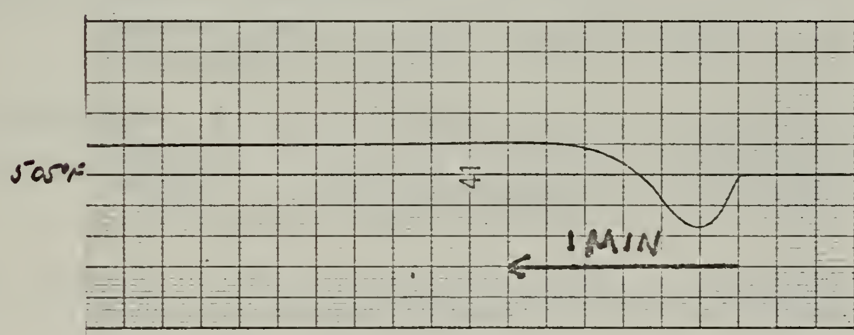
STEAM TEMPERATURE VERSUS TIME



T_F(MAX) = 593.8°F

T_F(SS) = 590.2°F

FUEL TEMPERATURE VERSUS TIME



T_{AV}(MIN) = 499.8°F

T_{AV}(SS) = 508.0°F

AVERAGE COOLANT TEMPERATURE VERSUS TIME

Fig. (D-4a). TIME RESPONSE OF POWER PLANT WITH EXTERNAL CONTROLLER.

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13. ABSTRACT

The dynamics of a proposed marine nuclear propulsion plant, under the condition of sudden changes in load, are studied by both an analog and digital simulation. Two inherent reactor characteristics, internal reactivity feedback and the existance of delayed neutron emitters, which effect power plant stability and controllability are considered in detail.

Comparison of the simulation results with practical data indicates that the simulation represented the dynamics of a marine nuclear power plant with a sufficient degree of accuracy to be extremely useful in the study of marine nuclear power plants.

Evaluation of the simulation results indicate that the proposed power plant is inherently stable when operating in the normal region. The desirability of control schemes are discussed and a control scheme utilizing a constant average coolant temperature program is implemented. This external control scheme significantly improved the response of the system.

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